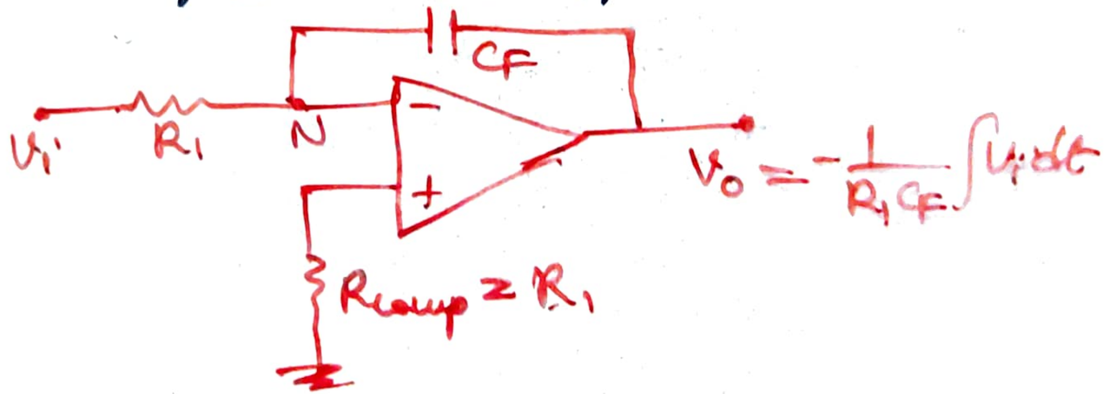
Integrator

→ Integrator provides an o/p voltage which is proportional to the time integral of the i/p



→ Node equation at node N is

$$\frac{v_i}{R_1} + C_F \frac{dv_o}{dt} = 0$$

$$\Rightarrow \frac{dv_o}{dt} = -\frac{1}{R_1 C_F} v_i$$

→ Integrating both sides, we get

$$\int_0^t dv_o = -\frac{1}{R_1 C_F} \int_0^t v_i\,dt$$

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$$\Rightarrow V_o(t) = -\frac{1}{R_1 C_F}\int_0^t V_i(t)\,dt + V_o(0)$$

where $V_o(0)$ is the initial o/p voltage

→ In freq. domain $V_o(S) = -\frac{1}{SR_1C_F} V_i(S)$

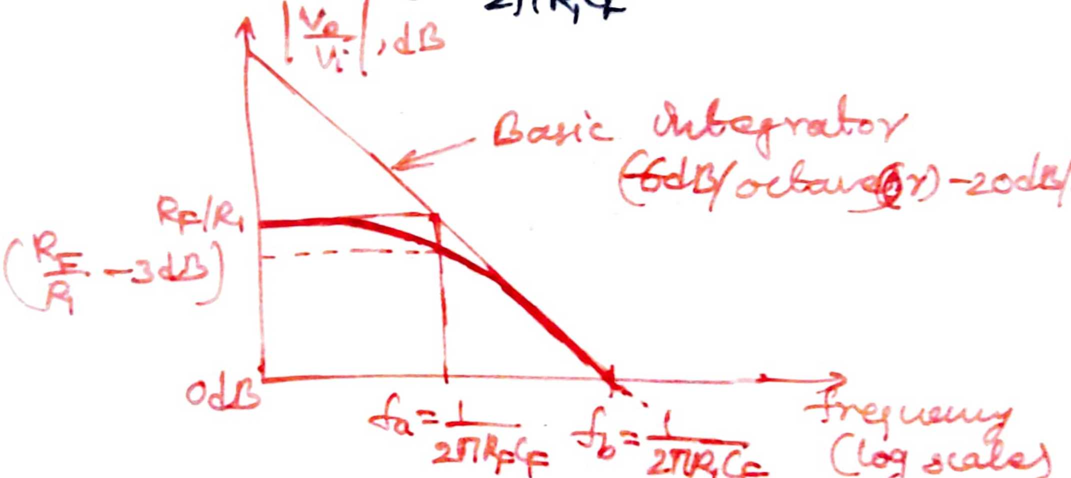
→ In steady state, $s = j\omega$ & we get

$$V_o(j\omega) = -\frac{1}{j\omega R_1 C_F} V_i(j\omega)$$

$$|A| = \left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \left|-\frac{1}{j\omega R_1 C_F}\right| = \frac{1}{\omega R_1 C_F}$$

$$= \frac{f_b}{f}$$

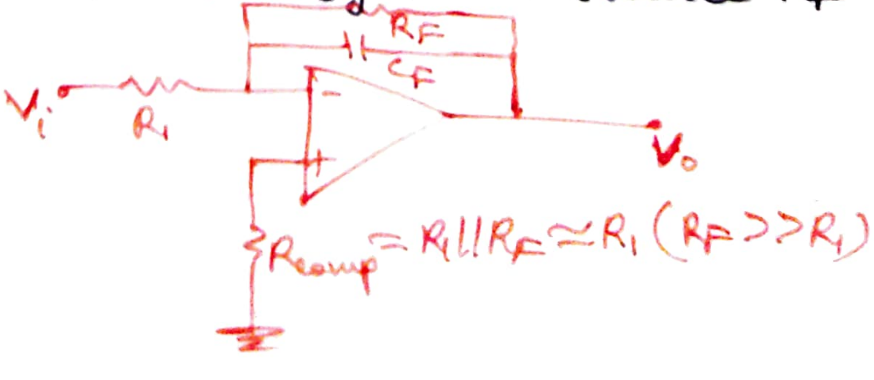
where $f_b = \frac{1}{2\pi R_1 C_F}$



→ At low freq.'s such as at dc ($\omega \equiv 0$), gain becomes infinite (or saturates)

Practical Integrator Circuit (Lossy Integrator)

→ The gain of an integrator at low freq. (dc) can be limited to avoid saturation problem if the feedback capacitor is shunted by a resistance R_F.



$$\frac{V_i(s)}{R_1} + sC_F V_o(s) + \frac{V_o(s)}{R_F} = 0$$

$$\Rightarrow \quad V_o(s) = -\frac{1}{sR_1C_F + R_1/R_F} V_i(s)$$

→ for $s = j\omega$

$$|A| = \left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{\omega^2 R_1^2 C_F^2 + R_1^2/R_F^2}} = \frac{R_F/R_1}{\sqrt{1+(\omega R_F C_F)^2}}$$

→ At low freq.'s gain is const. at R_F/R_1

→ The break freq. ($f = f_a$) at which gain is $0.707\,(R_F/R_1)$

$$\sqrt{1+(\omega R_F C_F)^2} = \sqrt{2}$$

→ Solving for $f = f_a$, $f_a = \frac{1}{2\pi R_F C_F}$

Q) ~~Design~~ Consider a practical integrator with $R_1 = 10k\Omega$, $R_f = 100k\Omega$, $C_f = 10nF$, determine the lower frequency limit of integration & study the response for i/ps (i) sine wave (ii) step i/p (iii) square wave.

Lower freq. limit of integration f_a is $f_a = \frac{1}{2\pi R_f C_f}$

$= 159 Hz$

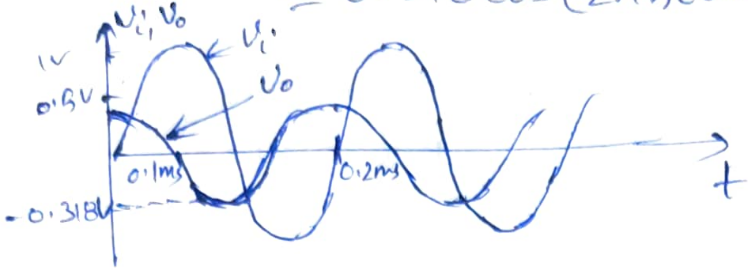
Sine wave i/p :- For an i/p of 1V peak sine wave at 5kHz the o/p V_o is

$$V_o(t) = -\frac{1}{R_1 C_f}\int V_i(t)\,dt$$

$$= -\frac{1}{10k \times 10n} \int 1 \sin(2\pi 5000t)\,dt$$

$$= -10^4 \int \sin(2\pi 5000t)\,dt = \frac{-10^4}{2\pi 5000}\left[-\cos(2\pi 5000t)\right]$$

$$= 0.318\cos(2\pi 5000t)$$



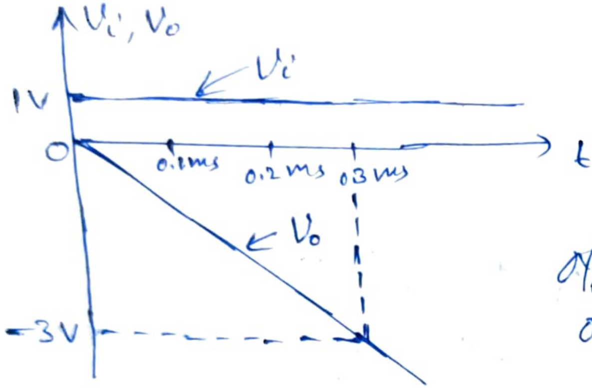
O/P is cosine wave with peak amplitude 0.318V

<u>Step i/p</u> If i/p is a step voltage $V_i = 1V$ for $0 \le t \le 0.3ms$ the o/p voltage is

$$V_o = -\frac{1}{R_1 C_f}\int_0^{0.3ms} 1\,dt$$

$$= \frac{-1}{10k \times 10n} \times t \Big|_{t=0}^{t=0.3ms}$$

$$= -3V$$

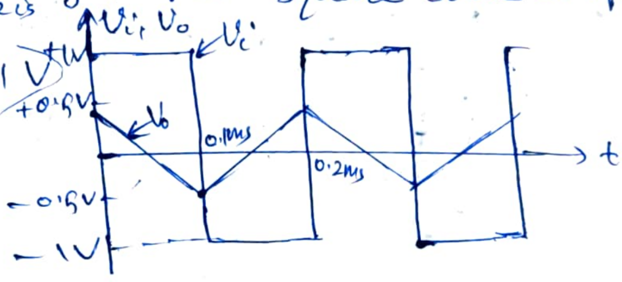


O/p is a ramp function with slope of 10V/ms

<u>Square wave i/p</u>

If the i/p waveform of 5kHz 1V peak square wave is applied. The peak value of o/p for first half cycle is

$$V_o = -\frac{1}{R_1 C_f}\int_0^{0.1ms} 1.\,dt = -1V$$



O/p is triangular wave

Q→ Design a practical integrator ckt with a dc gain of 10, to integrate a square wave of 10kHz

$$|A|_{dc} = \frac{R_f}{R_1} \Rightarrow 10 = \frac{R_f}{R_1}$$

i/p freq. is $f = 10kHz$

For proper integration $f \geq 10 f_a$ where f_a is break freq. of practical integrator

$$\frac{f}{f_a} \geq 10$$

$$f_a = \frac{f}{10} = \frac{10k}{10} = 1kHz$$

For practical integrator

$$f_a = \frac{1}{2\pi R_f C_f} \Rightarrow R_f C_f = 1.5915 \times 10^{-4}$$

Let: $R_1 = 10k\Omega$

$R_f = 10R_1 = 10 \times 10k = 100k\,\Omega$

$C_f = \dfrac{1.5915 \times 10^{-4}}{100 \times 10^3} = 1.5915 \times 10^{-9} F$

$\simeq 1.6\ nF$

$R_{comp} = R_1 || R_f = 9.09\ k\Omega$