- Complements of Numbers

→ Complements of numbers are used in digital computers to simplify the subtraction operation and for logical manipulation.

→ Simplifying operation leads to simpler, less expensive circuits to implement the operations.

→ There are two types of complements for each base-r system

1) r's complement 2) (r-1)'s complement

→ For binary numbers, the complements are 2's complement and 1's complement.

→ For decimal numbers, the complements are 10's complement and 9's complement.

Diminished Radix Complement [(r-1)'s complement]

→ Given a number N in base r having n digits, the (r-1)'s complement of N, i.e., its diminished radix complement, is defined as $(r^n-1)-N$.

→ For decimal numbers, r=10 and r-1=9, so the 9's complement of N is $(10^n-1)-N$.

10^n represents a number that consists of a single 1 followed by n 0's.

10^n-1 represents a number that consists of n 9's.

→ The 9's complement of 546700

$n=6 \quad 10^n=1000000$

$10^n-1=999999$

$$\begin{array}{r} 999999 \\ -\ 546700 \\ \hline 453299 \end{array}$$

→ The 9's complement of 012398 is.

$$\begin{array}{r} 999999 \\ -\ 012398 \\ \hline 987601 \end{array}$$

→ For binary numbers, r=2 and r-1=1, so the 1's complement of N is $(2^n-1)-N$.

2^n is represented by a binary number that consists of a 1 followed by n 0's.

2^n-1 is a binary number represented by n 1's

→ The 1's complement of 1011000

$n=7 \quad 2^n=(128)_{10}=(10000000)_2$

$2^n-1=1111111$

$$\begin{array}{r} 1111111 \\ -\ 1011000 \\ \hline 0100111 \end{array}$$

→ The 1's complement of 0101101 is

$$\begin{array}{r} 1111111 \\ -\ 0101101 \\ \hline 1010010 \end{array}$$

→ The 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's

→ The (r-1)'s complement of octal or hexadecimal numbers are obtained by subtracting each digit from 7 or F(15) respectively.

<u>Radix Complement [r's complement]</u>

→ The r's complement of an n-digit number N in base r is defined as $r^n - N$ for $N \neq 0$ and as 0 for $N = 0$.

→ The r's complement is obtained by adding 1 to the (r-1)'s complement, because $r^n - N = [(r^n - 1) - N] + 1$.

→ For decimal numbers, $r = 10$, so the 10's complement of number N is $10^n - N$ where 10^n represents a number that consists a 1 followed by n 0's

→ The 10's complement of 012398 is

$n = 6 \quad 10^n = 1000000$

$$\begin{array}{r} 1000000 \\ -\ 012398 \\ \hline 987602 \end{array}$$

→ The 10's complement of 246700 is

$$\begin{array}{r} 1000000 \\ 246700 \\ \hline 753300 \end{array}$$

→ For binary numbers, $r = 2$, so the 2's complement of N is $2^n - N$ where 2^n represents binary number that consists of 1 followed by n 0's.

→ The 2's complement of 1101100 is

$n = 7 \quad 2^n = (128)_{10} = (10000000)_2$

$$\begin{array}{r} 10000000 \\ -\ 1101100 \\ \hline 0010100 \end{array}$$

→ The 2's complement of 0110111 is

$$\begin{array}{r} 10000000 \\ 0110111 \\ \hline 1001001 \end{array}$$

→ If the number N contains a radix point, the point should be removed temporarily in order to form the r's or (r-1)'s complement.

→ The radix point is then restored to the complemented number in the same relative position.

→ The complement of the complement restores the number to its original value.

→ The r's complement of N is $r^n - N$, so that complement of the complement is $r^n - (r^n - N) = N$ and is equal to the original number.

Subtraction with Complements

→ The subtraction of two n-digit unsigned numbers M-N in base r can be done as follows:

1) Add the minuend M to the r's complement of the subtrahend N. Mathematically $M + (r^n - N) = M - N + r^n$.

2) If $M \geq N$, the sum will produce an end carry r^n, which can be discarded, the remaining is the result M-N.

3) If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N-M)$, which is the r's complement of (N-M). To obtain the answer, take the r's complement of the sum and place a negative sign in front.

→ Using 10's complement, subtract 72532 − 3250

M = 72532

10's complement of N = + 96750

Sum = 169282

Discard end carry of 10^5 = 100000 $(M \geq N)$

Answer = 69282

→ Using 10's complement, subtract 3250 − 72532

$$M = 03250$$
$$\text{10's complement of } N = +27468$$
$$\text{Sum} = 30718$$

There is no end carry, therefore the answer is −(10's complement of 30718) = −69282 (M<N)

→ Given two binary numbers X = 1010100 and Y = 1000011, perform the subraction (a) X−Y, and (b) Y−X by using 2's complement

(a)

$$X = 1010100$$
$$\text{2's Complement of } Y = 0111101$$
$$\text{Sum} = 10010001$$
$$\text{Discard end carry } 2^7 = 10000000$$
$$\text{Answer}(X-Y) = 0010001$$

(b)

$$Y = 1000011$$
$$\text{2's complement of } X = 0101100$$
$$\text{Sum} = 1101111$$

There is no end carry, therefore the answer is

$$Y-X = -(\text{2's complement of } 1101111)$$
$$= -0010001$$

→ Subtraction of unsigned numbers can also be done by means of the (r−1)'s complement.

→ Since the (r−1)'s complement is one less than the r's complement, the result of adding the minuend to the complement of the subtrahend produces a sum that is one less than the correct difference when an end carry occurs.

→ Removing the end carry and adding 1 to the sum is referred to as an end-around carry.

→ Using 1's complement find (a) X−Y and (b) Y−X where X = 1010100 and Y = 1000011

(a)

$$\begin{array}{rr} X = & 1010100 \\ \text{1's complement of } Y = + & 0111100 \\ \hline \text{Sum} = & 10010000 \\ \text{End-around carry} = + & 1 \\ \hline & 0010001 \end{array}$$

Answer $(X-Y) =$

(b)

$$\begin{array}{rr} Y = & 1000011 \\ \text{1's complement of } X = + & 0101011 \\ \hline \text{Sum} = & 1101110 \end{array}$$

→ There is no end carry, therefore the answer is

$Y - X = -$ (1's complement of 1101110) $= -0010001$

Signed Binary Numbers

→ Positive numbers (including zero) can be represented as unsigned numbers. However, to represent negative numbers, we need a notation.

→ In ordinary arithmetic, a negative number is indicated by a minus sign and a positive number by a plus sign.

→ Because of hardware limitations, computers must represent everything with binary digits.

→ It is customary to represent the sign with a bit placed in the leftmost position of the number. This representation of signed numbers is referred to as signed-magnitude convention

→ In this notation, the number consists of a magnitude and a symbol (+ or −) or a bit (0 or 1) indicating the sign.

→ When arithmetic operations are implemented in a computer, it is more convenient to use a different system, referred to as the signed-complement system for representing negative numbers.

→ The signed-magnitude system negates a number by changing its sign whereas the signed-complement system negates a number by taking its complement.

→ The signed-complement system can use either the 1's or 2's complement, but the 2's complement is the most common.

→ Consider the number 9, represented in binary with eight bits.

→ +9 is represented with a sign bit 0 in the left most position, followed by the binary equivalent of 9, which gives 00001001.

→ There are three different ways to represent −9 with eight bits:

signed-magnitude representation; 10001001

signed-1's-complement representation; 11110110

signed-2's-complement representation; 11110111

Decimal	signed-2's Complement	signed-1's Complement	signed Magnitude
+7	0111	0111	0111
+6	0110.	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

Arithmetic Addition

- The addition of two numbers in the signed-magnitude system follows the rules of ordinary arithmetic.
- If the signs are same, we add the two magnitudes and give the sum the common sign.
- If the signs are different, we subtract the smaller magnitude from the larger and give the (the) difference the sign of the larger magnitude.

  $(+25)+(-37) = -(37-25) = -12$

- In contrast, the rule for adding numbers in the signed-complement system does not require a comparision or subtraction, but only addition.
- The addition of two signed binary numbers with negative numbers represented in signed-2's -complement form is obtained from the addition of the two numbers, including their sign bits. A carry out of the sign-bit position is discarded.

+6	00000110		−6	11111010
+13	00001101		+13	00001101
+19	00010011		+7	1 00000111

+6	00000110		−6	11111010
−13	11110011		−13	11110011
−7	11111001		−19	1 11101101
	(2's complement of 7)			(2's complement of 19)

- Any carry out of the sign-bit position is discarded, and negative results are automatically in 2's-Complement form.
- In order to obtain a correct answer, we must ensure that the result has a sufficient number of bits to accomodate the sum
- If we start with two n-bit numbers and the sum occupies n+1 bits, we say that an overflow occurs.
- Overflow is a problem in computers because the number of bits that hold a number is finite, and a result that exceeds the finite value by 1 cannot be accomodated.

Arithmetic Subtraction

→ Subtraction of two signed binary numbers when negative numbers are in 2's-complement form is obtained by taking the 2's complement of the subtrahend (including the sign bit) and adding it to the minuend (including the sign bit).

→ A carry out of the sign-bit position is discarded.

→ This procedure is adopted because a subtraction operation can be changed to an addition operation if the sign of the subtrahend is changed.

$$(\pm A) - (+B) = (\pm A) + (-B)$$

$$(\pm A) - (-B) = (\pm A) + (+B)$$

$(-6) - (-13) = +7$

$$\begin{array}{rr} & -6 \\ - & -13 \\ \hline & +7 \end{array} \qquad \begin{array}{rr} & 11111010 \\ - & 11110011 \\ \hline & 00000111 \end{array} \qquad \begin{array}{rr} & 11111010 \\ + & 00001101 \\ \hline & \cancel{1}00000111 \end{array}$$

→ The binary numbers in the signed-complement system are added and subtracted by the same basic addition and subtraction rules as unsigned numbers.

→ Therefore, computers need only one common hardware circuit to handle both types of arithmetic.