- Number Systems

→ Digital systems have a prominent role in everyday life of the present technological period referred as the digital age.

→ One characteristic of digital system is their ability to represent and manipulate discrete elements of information.

→ Any set that is restricted to a finite number of elements contains discrete information.

For example, the 10 decimal digits, the 26 letters of the alphabet, the 52 playing cards, and the 64 squares of a chessboard are discrete sets.

→ Early digital computers were used for numeric computations. In this case, the discrete elements were the digits. From this application, the term digital computer emerged.

→ Discrete elements of information are represented in a digital system by physical quantities called signals. Electrical signals such as voltages and currents are the most common.

→ The signals in most present-day electronic digital systems use just two discrete values and are therefore said to be binary.

→ A binary digit, called a bit, has two values: 0 and 1.

→ Discrete elements of information are represented with groups of bits called binary codes.

→ Through various techniques, groups of bits can be made to represent discrete symbols, not necessarily numbers, which are then used to develop the system in a digital format.

→ Thus, a digital system is a system that manipulates discrete (symbols) elements of information represented internally in binary form.

→ There are many different number systems. In general, a number expressed in a base-r system has coefficients multiplied by powers of r.

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \ldots + a_2 \cdot r^2 + a_1 \cdot r + a_0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \ldots + a_{-m} \cdot r^{-m}$$

→ The coefficients a_j range in value from 0 to $r-1$.

→ To distinguish between numbers of different bases, we enclose the coefficients in paranthesis and write a subscript equal to the base used.

→ The most common number systems are:

1) Decimal number system
2) Binary number system
3) Octal number system
4) Hexadecimal number system

① Decimal Number System

→ The decimal number system is the most common and also most widely used for our daily numeric work.

→ The decimal number system is also called the base-10 or radix-10 system because it uses ten different symbols (the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) to represent any quantity.

→ Each digit in a decimal number represents a multiple of a power of 10.

$$(10.012)_{10} = 1\times10^{1} + 0\times10^{0} + 0\times10^{-1} + 1\times10^{-2} + 2\times10^{-3} = 10.012$$

$$(7392)_{10} = 7\times10^{3} + 3\times10^{2} + 9\times10^{1} + 2\times10^{0} = 7392$$

② Binary Number System

→ The binary number system, as the name suggests, deals with only two symbols, 0 or 1.

→ It is also called the base-2 or radix-2 system because it uses two digits (0, 1) to represent any quantity.

$$(1010)_2 = 1\times2^{3} + 0\times2^{2} + 1\times2^{1} + 0\times2^{0} = (10)_{10}$$

$$(100.01)_2 = 1\times2^{2} + 0\times2^{1} + 0\times2^{0} + 0\times2^{-1} + 1\times2^{-2} = (4.25)_{10}$$

③ Octal Number System

→ The octal number system deals with eight digits or symbols.

→ It is also called the base-8 or radix-8 system because it uses eight different symbols or digits (0, 1, 2, 3, 4, 5, 6, 7) to represent any quantity.

$$(127.4)_8 = 1\times8^{2} + 2\times8^{1} + 7\times8^{0} + 4\times8^{-1} = (87.5)_{10}$$

$$(231)_8 = 2\times8^{2} + 3\times8^{1} + 1\times8^{0} = (153)_{10}$$

④ Hexadecimal Number System

→ The hexadecimal number system deals with sixteen digits or symbols.

→ It is also called the base-16 or radix-16 system because it uses sixteen different symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F) to represent any quantity.

→ The letters of the alphabet are used to supplement the 10 decimal digits when the base of the number is greater than 10

→ The letters A, B, C, D, E, F are used for the digits 10, 11, 12, 13, 14, 15 respectively.

$$(B65F)_{16} = 11\times16^3 + 6\times16^2 + 5\times16^1 + 15\times16^0 = (46687)_{10}$$

$$(A.B10)_{16} = 10\times16^0 + 11\times16^{-1} + 1\times16^{-2} + 0\times16^{-3} = (10.6914)_{10}$$

- **Base Conversion Methods**

→ The conversion of a number in base r to decimal is done by expanding the number in a power series of radix r and adding all the terms.

$$(110101)_2 = 1\times2^5 + 1\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$$
$$= (53)_{10}$$

$$(3071)_8 = 3\times8^3 + 0\times8^2 + 7\times8^1 + 1\times8^0$$
$$= (1593)_{10}$$

$$(1234)_{16} = 1\times16^3 + 2\times16^2 + 3\times16^1 + 4\times16^0$$
$$= (4660)_{10}$$

$$(1010.11001)_2 = 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0 + 1\times2^{-1} + 1\times2^{-2}$$
$$+ 0\times2^{-3} + 0\times2^{-4} + 1\times2^{-5}$$
$$= (10.7813)_{10}$$

$$(2071.31)_8 = 2\times8^3 + 0\times8^2 + 7\times8^1 + 1\times8^0 + 3\times8^{-1}$$
$$+ 1\times8^{-2}$$
$$= (1081.3906)_{10}$$

$$(ABC.DE)_{16} = 10\times16^2 + 11\times16^1 + 12\times16^0 + 13\times16^{-1}$$
$$+ 14\times16^{-2}$$
$$= (2748.8672)_{10}$$

→ There is a general procedure for the reverse operation of converting a decimal number to a number in base r.

→ If the number includes a radin point, it is necessary to seperate the number into an integer part and a fraction part, since each part must be converted differently.

→ The conversion of a decimal integer to a number in base r is done by dividing the number and all successive quotients by r and accumulating the remainders.

→ The fraction is multiplied by radin r to give an integer and a fraction. Then the new fraction is multiplied by r to give a new integer and a new fraction

→ The process is continued until the fraction becomes 0 or until the number of digits has sufficient accuracy.

→ Convert decimal 41 to binary

we divide the given decimal number successively by 2 and read the remainders upwards to get the equivalent binary number.

Successive division		Remainder
2	41	
2	20	1
2	10	0
2	5	0
2	2	1
2	1	0
	0	1 ↑

$(41)_{10} = (101001)_2$

→ Convert $(0.75)_{10}$ to binary

we multiply the given decimal fraction by 2 to give an integer and a fraction. Then the new fraction is multiplied by 2 to give a new integer and a new fraction.

The process is continued until the fraction becomes 0 or until the number of digits has sufficient accuracy reading the integers from top to bottom.

	Integer
$0.75 \times 2 = 1.50$	1
$0.50 \times 2 = 1.00$	1

$(0.75)_{10} = (0.11)_2$

→ Convert $(105.15)_{10}$ to binary

Conversion of integer 105

		Remainder
2	105	
2	52	1
2	26	0
2	13	0
2	6	1
2	3	0
2	1	1
	0	1

$(105)_{10} = (1101001)_2$

Conversion of fraction 0.15

	Integer
$0.15 \times 2 = 0.30$	0
$0.30 \times 2 = 0.60$	0
$0.60 \times 2 = 1.20$	1
$0.20 \times 2 = 0.40$	0
$0.40 \times 2 = 0.80$	0
$0.80 \times 2 = 1.60$	1

$(0.15)_{10} = (0.001001)_2$

This particular fraction can never be expressed exactly in binary. This process may be terminated after few steps.

$\therefore (105.15)_{10} = (1101001.001001)_2$

→ Convert $(41)_{10}$ to an octal number

8	41	
8	5	1
	0	5

$(41)_{10} = (51)_8$

→ Convert $(0.513)_{10}$ to octal

$(0.513)_{10} = (0.406517..)_8$

$0.513 \times 8 = 4.104$	4
$0.104 \times 8 = 0.832$	0
$0.832 \times 8 = 6.656$	6
$0.656 \times 8 = 5.248$	5
$0.248 \times 8 = 1.984$	1
$0.984 \times 8 = 7.872$	7

→ Convert decimal number 200.60 to octal

$$\begin{array}{r|l} 8 & 200 \\ \hline 8 & 25 - 0 \\ \hline 8 & 3 - 1 \\ \hline & 0 - 3 \end{array} \uparrow$$

$(200)_{10} = (310)_8$

$0.60 \times 8 = 4.8$ 4

$0.8 \times 8 = 6.4$ 6

$0.4 \times 8 = 3.2$ 3

$0.2 \times 8 = 1.6$ 1

$0.6 \times 8 = 4.8$ 4

$\downarrow (0.60)_{10} = (0.46314\ldots)_8$

$(200.60)_{10} = (310.46314)_8$

→ Convert $(23469)_{10}$ to a hexadecimal number

$$\begin{array}{r|l} 16 & 23469 \\ \hline 16 & 1466 - 13\,(D) \\ \hline 16 & 91 - 10\,(A) \\ \hline 16 & 5 - 11\,(B) \\ \hline & 0 - 5 \end{array} \uparrow$$

$(23469)_{10} = (5BAD)_{16}$

→ Convert $(0.6566)_{10}$ to a hexadecimal number

$0.6566 \times 16 = 10.5056$ 10(A) $\downarrow$

$0.5056 \times 16 = 8.0896$ 8

$0.0896 \times 16 = 1.4336$ 1

$0.4336 \times 16 = 6.9376$ 6

$0.9376 \times 16 = 15.0016$ 15(F)

$(0.6566)_{10} = (0.A816F)_{16}$

(up to 5 decimal places)

→ Convert $(250.225)_{10}$ to a hexadecimal number

$$\begin{array}{r|l} 16 & 250 \\ \hline 16 & 15 - 10\,(A) \\ \hline & 0 - 15\,(F) \end{array} \uparrow$$

$(250)_{10} = (FA)_{16}$

$0.225 \times 16 = 3.6$ 3

$0.6 \times 16 = 9.6$ 9

$0.6 \times 16 = 9.6$ 9

$0.6 \times 16 = 9.6$ 9

$0.6 \times 16 = 9.6$ 9

$(0.225)_{10} = (0.39999)_{16}$

$(250.225)_{10} = (FA.39999)_{16}$

→ The conversion from and to binary, ~~and to~~ octal and hexadecimal plays an important role in digital computers, because shorter patterns of hex characters are easier to recognize than long patterns of 1's and 0's.

→ Since $2^3 = 8$ and $2^4 = 16$, each octal digit corresponds to three binary digits and each hexadecimal digit corresponds to four binary digits.

Binary to Octal Conversion

→ To convert a binary number to an octal number, starting from the binary point make groups of 3 bits each, on either side of the binary point, and replace each 3-bit binary group by the equivalent octal digit.

→ Convert $(110101.101010)_2$ to octal

$$\underbrace{110}_{6}\ \underbrace{101}_{5}\,.\,\underbrace{101}_{5}\ \underbrace{010}_{2}$$

$(110101.101010)_2 = (65.52)_8$

→ Convert $(10101111001.0111)_2$ to octal

$$\underbrace{010}_{2}\ \underbrace{101}_{5}\ \underbrace{111}_{7}\ \underbrace{001}_{1}\,.\,\underbrace{011}_{3}\ \underbrace{100}_{4}$$

$(10101111001.0111)_2 = (2571.34)_8$

Octal to Binary Conversion

→ To convert a given octal number to a binary, just replace each octal digit bytes by its 3-bit binary equivalent.

→ Convert $(367.52)_8$ to binary.

3 6 7 . 5 2

011 110 111 . 101 010

$(367.52)_8 = (011110111.101010)_2$

Binary to Hexadecimal Conversion

→ To convert a binary number to hexadecimal number, starting from the binary point, make groups of 4 bits each, on either side of the binary point and replace each 4bit group by the equivalent hexadecimal digit.

→ Convert $(1011011011)_2$ to hexadecimal

$$\underbrace{0\,0\,1\,0}_{2}\quad \underbrace{1\,1\,0\,1}_{D}\quad \underbrace{1\,0\,1\,1}_{B} \qquad (1011011011)_2 = (2DB)_{16}.$$

→ Convert $(01011111011.011111)_2$ to hexadecimal

$$\underbrace{0\,0\,1\,0}_{2}\quad \underbrace{1\,1\,1\,1}_{F}\quad \underbrace{1\,0\,1\,1}_{B}\ .\ \underbrace{0\,1\,1\,1}_{7}\quad \underbrace{1\,1\,0\,0}_{C}$$

$$(01011111011.011111)_2 = (2FB.7C)_{16}$$

Hexadecimal to Binary Conversion

→ To convert a hexadecimal number to binary, replace each hex digit by its 4-bit binary group.

→ Convert $(4BAC)_{16}$ to binary

$$\begin{array}{cccc} 4 & B & A & C \\ 0100 & 1011 & 1010 & 1100 \end{array} \qquad (4BAC)_{16} = (0100101110101100)_2$$

→ Convert $(3A9E.B0D)_{16}$ to binary

$$\begin{array}{cccccccc} 3 & A & 9 & E & . & B & 0 & D \\ 0011 & 1010 & 1001 & 1110 & . & 1011 & 0000 & 1101 \end{array}$$

$$(3A9E.B0D)_{16} = (0011101010011110.101100001101)_2$$