Binary Adder-Subtractor

~~Half Adder~~

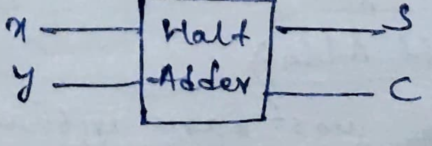
→ The most basic arithmetic operation is the addition of two binary digits.

- → This simple addition consists of four possible elementary operations: 0 + 0 = 0, 0 + 1 = 1, 1 + 0 = 1, and 1 + 1 = 10.
- → The first three operations produce a sum of one digit, but when both augend and addend bits are equal to 1, the binary sum (~~of one~~) consists of two digits.
- → The higher significant bit of this result is called a carry.
- → A combinational circuit that performs the addition of two bits is called a half-adder.
- → One that performs the addition of three bits (two significant bits and a previous carry) is a full adder.
- → A binary adder-subtractor is a combinational circuit that performs the arithmetic operations of addition and subtraction with binary numbers.
- → We will develop this circuit by means of a hierarchial design.
- → The half adder design is carried out first, from which we develop the full adder.
- → Connecting n full adders in cascade produces a binary adder for two n-bit numbers
- → The subtraction circuit is included in a complementing circuit.

Half Adder

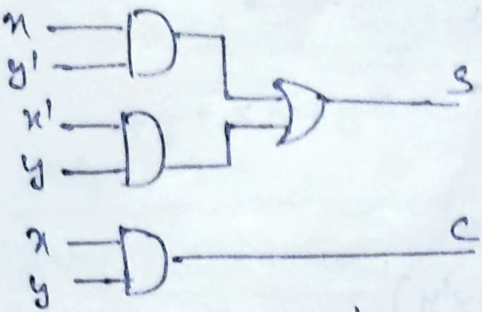
- → This circuit needs two binary inputs and two binary outputs.
- → The input variables designate the augend and addend bits; the output variables produce the sum and carry.
- → We assign symbols x and y to the two inputs and S (for sum) and C (for carry) to the outputs.

x	y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



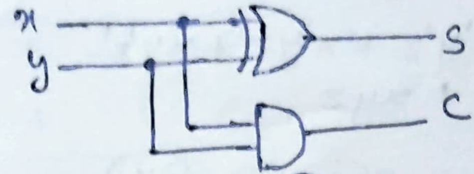
$S = x'y + xy'$

$C = xy$



$s = xy' + x'y$

$c = xy$



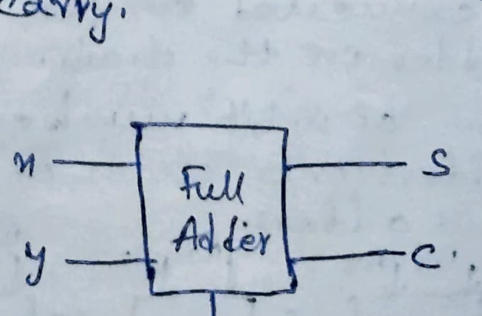
$S = x \oplus y$

$C = xy$

Full Adder

- Addition of n-bit binary numbers requires the use of a full adder, and the process of addition proceeds on a bit-by-bit basis, right to left, begining with the least significant bit.
- After the least significant bit, addition at each position adds not only the respective bits of the words, but also consider a possible carry bit from addition at the previous position.
- A full adder is a combinational circuit that forms the arithmetic sum of three bits.
- It consists of three inputs and two outputs
- Two of the input variables, denoted by x and y, represent the two significant bits to be added. The third input, z represents the carry from the previous lower significant position.
- The two outputs are designated by the symbols S for sum and C for carry.

x	y	z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



$x \backslash yz$	00	01	11	10
0	m_0	m_1 1	m_3	m_2 1
1	m_4 1	m_5	m_7 1	m_6

$x \backslash yz$	00	01	11	10
0	m_0	m_1	m_3 1	m_2
1	m_4	m_5 1	m_7 1	m_6 1

$S = x'y'z + x'yz' + xy'z' + xyz$

$C = xy + xz + yz$

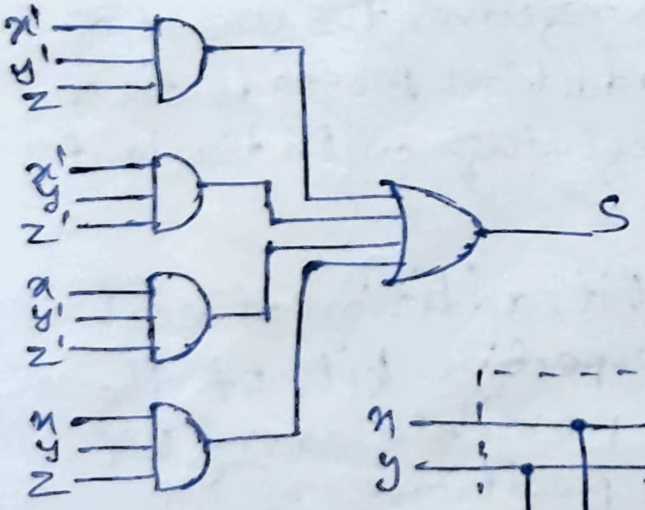
$S = z \oplus (x \oplus y)$

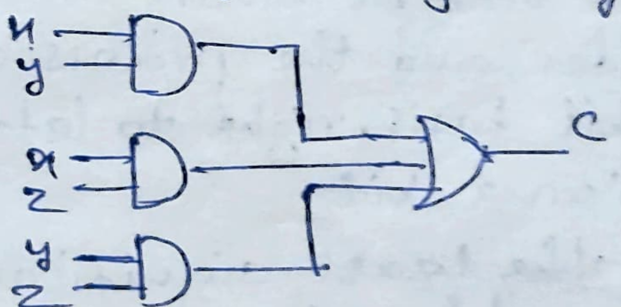
$= z'(xy' + x'y) + z(xy' + x'y)'$

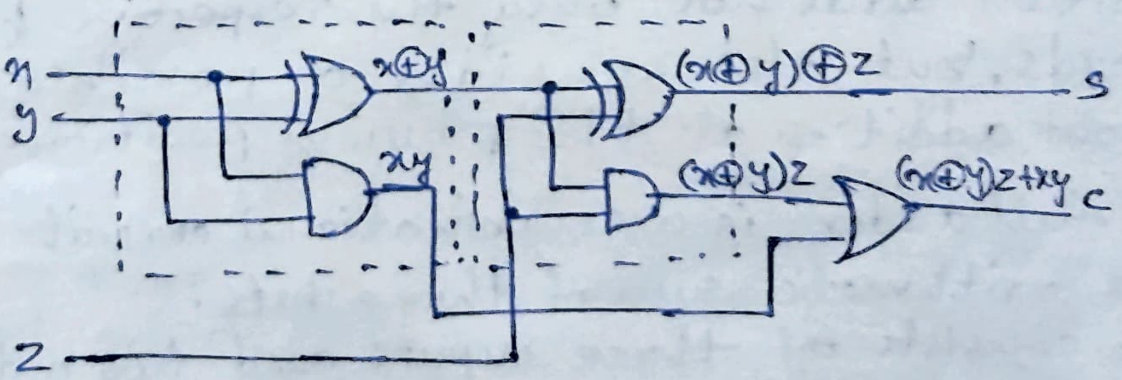
$= z'(xy' + x'y) + z(xy + x'y')$

$= xy'z' + x'yz' + xyz + x'y'z$

$C = z(xy' + x'y) + xy = xy'z + x'yz + xy$





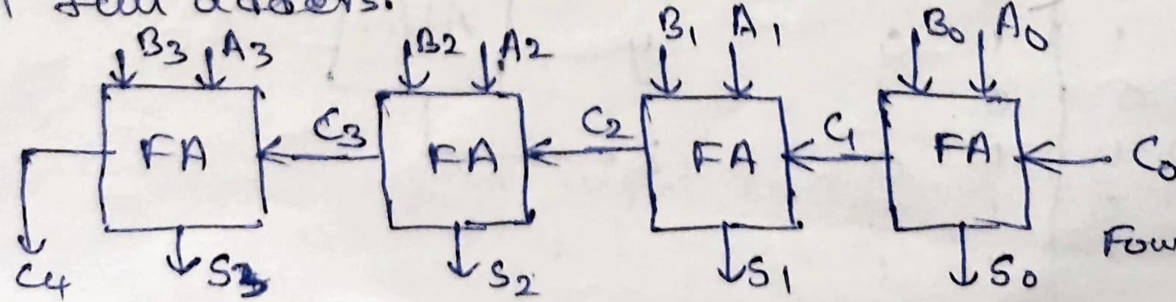


Binary Adder

→ A binary adder is a digital circuit that produces the arithmetic sum of two binary numbers.

→ It can be constructed with full adders connected in cascade, with the output carry from each full adder connected to the input carry of the next full adder in the chain.

→ Addition of n-bit numbers requires a chain of n full adders or a chain of one-half adder and n-1 full adders.



Four-bit adder

Subscript i:	3	2	1	0	
Input carry	0	1	1	0	C_i
Augend	[illegible]	0	1	1	A_i
Addend	0	0	1	1	B_i
Sum	1	1	1	0	S_i
Output Carry	0	0	1	1	C_{i+1}