

UNIT - INumber Systems* Review of Number Systems:

- In general, in any number system there is an ordered set of symbols known as digits with rules defined for performing arithmetic operations like addition, multiplication, etc.
- A collection of these digits makes a number which in general has two parts - integer and fractional, set apart by a radix point (.), that is

Mixed no.

$$(N)_b = \underbrace{d_{n-1} d_{n-2} \dots d_i \dots d_1 d_0}_{\text{Integer portion}} \cdot \underbrace{d_{-1} d_{-2} \dots d_{-f} \dots d_{-m}}_{\text{Fractional portion}}$$

↑
Radix Point

Where $N \rightsquigarrow$ a number

$b \rightsquigarrow$ radix or base of the number system

$n \rightsquigarrow$ no. of digits in integer portion

$m \rightsquigarrow$ no. of digits in fractional portion

$d_{n-1} \rightsquigarrow$ Most significant bit (MSB)

$d_{-m} \rightsquigarrow$ least significant bit (LSB)

and $0 \leq (d_i \text{ or } d_{-f}) \leq b-1$

1. Decimal Number System:

- Decimal system contains ten unique symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
- Base or radix is ten. Radix \rightarrow indicates the no. of unique digits.
- The value of a decimal number is the sum of the products of the digits of that number with their respective column weights.

EX: ① $7,392 = 7 \times 1000 + 3 \times 100 + 9 \times 10 + 2$
 $= 7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$

② $9256.26 = 9 \times 1000 + 2 \times 100 + 5 \times 10 + 6 + 2 \times 10^{-1} + 6 \times 10^{-2}$
 $= 9 \times 10^3 + 2 \times 10^2 + 5 \times 10 + 6 \times 10^0 + 2 \times 10^{-1} + 6 \times 10^{-2}$

* 9's and 10's complements:

- Subtraction of decimal numbers can be accomplished by 9's and 10's complement methods.

9's complement:

\rightarrow 9's complement of a decimal no. is obtained by subtracting each digit of that decimal no from 9.

EX: Find 9's complement of following decimal nos

(a) 3465 (b) 782.54 (c) 4526.075

Sol: (a)
$$\begin{array}{r} 9999 \\ - 3465 \\ \hline 6534 \checkmark \end{array}$$

(b)
$$\begin{array}{r} 999.99 \\ - 782.54 \\ \hline 217.45 \checkmark \end{array}$$

(c)
$$\begin{array}{r} 9999.999 \\ - 4526.075 \\ \hline 5473.924 \checkmark \end{array}$$

10's Complement:

↳ 10's Complement of a decimal no. is obtained by adding a 1 to its 9's Complement

Ex: Find 10's Complement of the following decimal

(a) nos. 4069

(b) 1056.074

Sol: (a)
$$\begin{array}{r} 9999 \\ - 4069 \\ \hline 5930 \\ + 1 \\ \hline 5931 \checkmark \end{array}$$

(b)
$$\begin{array}{r} 9999.999 \\ 1056.074 \\ \hline 8943.925 \\ + 1 \\ \hline 8943.926 \checkmark \end{array}$$

9's complement Method of subtraction:

Step 1: Obtain 9's complement of the subtrahend.

Step 2: Add ~~the~~ it to the minuend, call this no. as intermediate result.

Step 3: \hookrightarrow If there is a carry, it indicates that the answer is positive. Add the carry to LSB of this result to get the answer. This is called end around carry.

Step 4: If there is no carry, it indicates that the answer is negative and the intermediate result is its 9's complement. Take 9's complement of this result & place a negative sign in front to get the answer.

Ex: Subtract the following nos. using the 9's complement method.

(a) $745.81 - 436.62$ (b) $436.62 - 745.81$

Sol: (a)
$$\begin{array}{r} 745.81 \\ - 436.62 \\ \hline 309.19 \end{array} \Rightarrow \begin{array}{r} 745.81 \\ + 563.37 \text{ (9's complement of 436.62)} \\ \hline \textcircled{1} 309.18 \text{ (Intermediate result)} \\ \hookrightarrow + \quad 1 \text{ (End around carry)} \\ \hline 309.19 \text{ (Answer)} \end{array}$$

(3)

$$\begin{array}{r}
 (b) \quad 436.62 \\
 - 745.81 \\
 \hline
 - 309.19 \\
 \hline
 \end{array}
 \Rightarrow
 \begin{array}{r}
 436.62 \\
 + 254.18 \text{ (9's complement of 745.81)} \\
 \hline
 690.80 \text{ (intermediate result with no carry)} \\
 \hline
 \end{array}$$

↓ 9's complement

$$- 309.19 \text{ (Answer)}$$

10's Complement Method of Subtraction:

Step 1: obtain 10's complement of the subtrahend.

Step 2: Add it to the minuend.

Step 3: If there is a carry, ignore it. The presence

of the carry indicates that the answer is positive. The result obtained is itself the answer.

Step 4: If there is no carry, it indicates that the answer is negative & the result obtained is its 10's complement. Take 10's complement of the result & place a negative sign in front to get the answer.

Ex: Subtract the following numbers using the 10's complement method.

(a) $2928.54 - 416.73$

(b) $416.73 - 2928.54$

Sol: (a)
$$\begin{array}{r} 2928.54 \\ - 0416.73 \\ \hline 2511.81 \end{array} \Rightarrow \begin{array}{r} 2928.54 \\ + 9583.27 \text{ (10's complement of } 416.73) \\ \hline \textcircled{1} 2511.81 \\ \downarrow \\ \text{Answer} \end{array}$$
 (ignore carry)

(b)
$$\begin{array}{r} 0416.73 \\ - 2928.54 \\ \hline -2511.81 \end{array} \Rightarrow \begin{array}{r} 0416.73 \\ + 7071.46 \text{ (10's complement of } 2928.54) \\ \hline 7488.19 \text{ (No carry)} \\ \downarrow \text{ Take 10's complement} \\ -2511.81 \text{ Answer} \end{array}$$

2. Binary Number system:

- A binary no. consists of a sequence of bits, each of which is either a 0 or a 1.
- Base or radix is two.
- The binary number system is used in digital computers because the switching circuits used in these computers use two-state devices such as transistors, diodes etc.
- A transistor can be OFF or ON, a switch can be OPEN or CLOSED, a diode can be OFF or ON, etc. These devices have to exist in one of the two possible states. i.e; 0 and 1.

→ Binary to Decimal Conversion:

Examples:

① Convert 10101_2 to decimal.

Sol:
$$\begin{matrix} 1 & 0 & 1 & 0 & 1 \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{matrix} = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$

$$= 16 + 0 + 4 + 0 + 1$$

$$= 21_{10}$$

② Convert 11011.101_2 to decimal

Sol:
$$\begin{matrix} 1 & 1 & 0 & 1 & 1 & . & 1 & 0 & 1 \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 & & 2^{-1} & 2^{-2} & 2^{-3} \end{matrix}$$

$$= (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3})$$

$$= 16 + 8 + 0 + 2 + 1 + 0.5 + 0 + 0.125$$

$$= 27.625_{10}$$

③ Convert 1001011_2 to decimal

Sol:
$$(1 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

$$= 64 + 0 + 0 + 8 + 0 + 2 + 1$$

$$= 75_{10}$$

→ Decimal to Binary Conversion:

Method 1: Sum-of-weights

Ex: ① Convert 163_{10} to binary using sum-of-weights method.

Sol:
→ The largest ^{decimal} no., which is a power of 2, not exceeding 163 is 128.

$$128 = 2^7 = 10000000_2$$

The remainder is $163 - 128 = 35$

→ The largest no., which is a power of 2, not exceeding

35 is 32

$$32 = 2^5 = 100000_2$$

The remainder is $35 - 32 = 3$

→ The largest no., which is a power of 2, not exceeding

3 is 2

$$2 = 2^1 = 10_2$$

The remainder is $3 - 2 = 1 \rightarrow 2^0 = 1_2$

$$\begin{aligned} \therefore 163_{10} &= 100000000_2 + 100000_2 + 10_2 + 1_2 \\ &= 10100011_2 \end{aligned}$$

Repeat the above process on the successive remainders till you get a 0 remainder

Method 2: Double-dabble method.

Convert
EX: ① 163_{10} to binary

Successive division

2		163
2		81
2		40
2		20
2		10
2		5
2		2
2		1
		0

Remainder

1
1
0
0
0
1
0
1
↑
MSB

Dividend 163 (81 → quotient)
divisor 2
 $\begin{array}{r} 2 \overline{) 163} \\ \underline{162} \\ 1 \end{array}$ remainder

Successive
divisor by 2
till quotient
becomes zero.

∴ Therefore

$$163_{10} = (10100011)_2$$

More
Examples:

② Convert 163.875_{10} to binary

Method 1: Integer part
 $163_{10} = 10100011_2$

Fraction part 0.875_{10}

→ The largest fraction, which is a power of 2,

not exceeding 0.875 is 0.5

$$0.5 = 2^{-1} = 0.100_2$$

The remainder is $0.875 - 0.5 = 0.375$

→ The largest fraction, which is a power of 2, not exceeding 0.375 is 0.25

$$0.25 = 2^{-2} = 0.01_2$$

The remainder is $0.375 - 0.25 = 0.125$

→ The largest fraction, which is a power of 2, not exceeding 0.125 is 0.125 itself

$$0.125 = 2^{-3} = 0.001_2$$

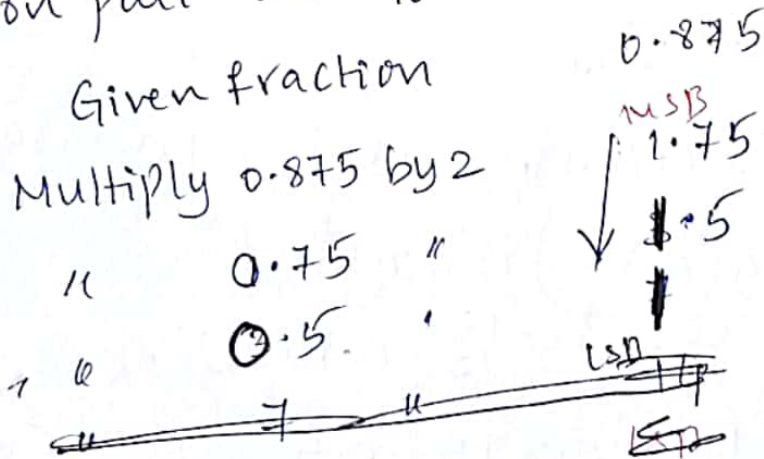
$$\therefore 0.875_{10} = 0.100_2 + 0.01_2 + 0.001_2 = 0.111_2$$

The final result is $163.875_{10} = 10100011.111_2$

Method 2:

Integer part $163_{10} = 10100011$

Fraction part 0.875_{10}



$$(0.875)_{10} = (0.111)_2$$

Successive multiplication by 2

or till the desired accuracy is obtained.

$$\therefore 163.875_{10} = 10100011.111_2$$

Decimal fraction are successively multiplied by 2, till the fraction part of the product is 0

③ Convert decimal 41 to binary

⑥

Sol: Successive division

2	41	Remainder
2	20	1 LSB
2	10	0
2	5	0
2	2	1
2	1	0
	0	1 ↑

MSB

$$(41)_{10} = (101001)_2$$

④ Convert $(0.6875)_{10}$ to binary

Sol: Sol:

Given fraction: 0.6875

Multiply 0.6875 by 2	LSB	1.375
" 0.375 "	↓	0.75
" 0.75 "		1.5
" 0.5 "		1.0

MSB

$$(0.6875)_{10} = (0.1011)_2$$

* Binary Addition:

The rules for binary addition are

$$0+0=0; 0+1=1; 1+0=1; 1+1=10$$

ie, 0 with a carry 1

Ex: Add the binary nos. 1101.101 and 11.011

Sol:

$$\begin{array}{r} 1101.101 \\ + 111.011 \\ \hline 10101.000 \end{array}$$

* Binary subtraction:

The rules for binary subtraction are

$0-0=0$; $1-1=0$; $1-0=1$; $0-1=1$, with a borrow of 1.

Ex: Subtract 111.111_2 from 1010.01_2

Sol:

$$\begin{array}{r} 1010.010 \\ - 111.111 \\ \hline 011.011 \end{array}$$

* Binary Multiplication:

The rules for binary multiplication are

$0 \times 0 = 0$; $1 \times 1 = 1$; $1 \times 0 = 0$; $0 \times 1 = 0$

3. Octal Number System:

- Octal number system was extensively used by early minicomputers.
- It is a positional weighted system.
- Its base or radix is 8.
- It has 8 independent symbols 0, 1, 2, 3, 4, 5, 6 and 7.
- $\because 8 = 2^3$, every 3-bit group of binary can be represented by an octal digit.
- When dealing with large binary numbers of many bits, it is convenient and more efficient for us to write the numbers in octal rather than binary.

Octal to Binary Conversion:

→ Just replace each octal digit by its 3-bit binary equivalent

Ex: Convert 367.52_8 to binary.

Sol: ①

3	6	7	.	5	2
011	110	111		101	010

Result $\Rightarrow 011110111.101010_2$

Ex: Convert 723.143_8 to binary

Sol: ②

7	2	3	.	1	4	3
111	010	011		001	100	011

Result $\Rightarrow 111010011.001100011_2$

Binary to Octal Conversion:

- For given binary number, make groups of 3 bits each. on either side of the binary point
- Replace each 3-bit binary group by the equivalent octal digit

EX: ① Convert 110101.101010_2 to octal

Sol:
110 101 . 101 010
6 5 5 2 Result $\Rightarrow 65.52_8$

EX: ② Convert 10101111001.0111_2 to octal

Sol: 010 101 111 001 . 011 100
2 5 7 1 . 3 4

Result $\Rightarrow 2571.34_8$

Octal to Decimal Conversion:

- Multiply each digit in the octal no. by the weight of its position and add all the product terms.

EX: ① Convert 4057.06_8 to decimal

Sol:
 $4057.06_8 = 4 \times 8^3 + 0 \times 8^2 + 5 \times 8^1 + 7 \times 8^0 + 0 \times 8^{-1} + 6 \times 8^{-2}$
 $= 2048 + 0 + 40 + 7 + 0 + 0.0937$
 $= 2095.0937_{10}$

Ex: ② Convert 5497_8 to decimal

Sol:
$$\begin{array}{cccc} 5 & 4 & 9 & 7 \\ 8^3 & 8^2 & 8^1 & 8^0 \end{array} = 5 \times 8^3 + 4 \times 8^2 + 9 \times 8 + 7 \times 8^0$$

Decimal to Octal Conversion:

→ Convert integer and fraction parts separately.

→ To convert given decimal no.

↳ Divide the given no. by 8 till quotient is 0.

↳ Last remainder is MSD.

→ To convert given decimal fraction,

↳ Successively multiply it by 8 till product is 0.

or till required accuracy is obtained

↳ The first integer from the top is MSD.

Ex: ① Convert 378.93_{10} to octal

Sol:

Integer
conversion

Successive division

$$\begin{array}{r|l} 8 & 378 \\ \hline & 47 \\ 8 & \overline{47} \\ & 5 \\ 8 & \overline{5} \\ & 0 \end{array}$$

Remainders

2

7

↑ 5

$\therefore 378_{10} = 572_8$

3

Fraction Conversion: Successive multiplication

0.93×8	\downarrow 7.44
0.44×8	3.52
0.52×8	4.16
0.16×8	1.28

$\therefore 0.93_{10} = 0.734_8$

Final result $\Rightarrow 378.93_{10} = 572.734_8$

EX: Convert 5497_{10} to binary.
 (2) Conversion

Sol: \uparrow Large decimals to binary and large binary nos to decimal can be conveniently and quickly performed via Octal

$5497_{10} \rightarrow$ OCTAL \rightarrow BINARY

Successive Division

8	5497
8	687
8	85
8	10
8	1
	0

Remainders

- 1
- 7
- 5
- 2
- 1

$\therefore 5497_{10} = 12571_8 = 001010101111001_2$

$\begin{matrix} / & / & / & / \\ 001 & 010 & 101 & 111 & 001 \end{matrix}$

EX: ③ Convert 101111010001_2 to decimal.

⑨

Sol: Binary \rightarrow OCTAL \rightarrow DECIMAL

$$\begin{array}{cccc} 101 & 111 & 010 & 001 \\ 5 & 7 & 2 & 1 \end{array} \quad 5721_8$$

\downarrow

$$8 \cdot 5 \times 8^3 + 7 \times 8^2 + 2 \times 8^1 + 1 \times 8^0$$
$$= 2560 + 448 + 16 + 1$$
$$= 3025_{10}$$

4. Hexadecimal Number System:

- It is a positional weighted system.
- It has 16 unique symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, A, B, C, D, E and F
- Base or Radix is 16.
- A 4-bit group is called a nibble.
- Since computer words come in 8 bits, 16 bits, 32 bits and so on, i.e.; multiples of 4 bits, they can be easily represented in hexadecimal
- This no. system is particularly useful for human communications with computers.
- This is most commonly used no. system in computer literature. It is used in large and small computers

Hexadecimal to Binary Conversion:

→ Replace each hex digit by its 4-bit binary group.

EX: (1) Convert $4BAC_{16}$ to binary

Sol:

4 B A C
0100 1011 1010 1100

Result \Rightarrow 0100101110101100₂

EX: (2) Convert $3A9E.B0D_{16}$ to binary.

Sol:

3 A 9 E . B 0 D
0011 1010 1001 1110 1011 0000 1101

Result \Rightarrow 0011101010011110.101100001101₂

Binary to Hexadecimal Conversion:

→ Make groups of 4 bits each on either side of the binary point and replace each 4-bit group by the equivalent hexadecimal digit

EX: (1) Convert 1011011011_2 to hexadecimal

Sol:

10 1101 1011
↓ ↓ ↓
2 D B

Result \Rightarrow 2DB₁₆

EX: ② Convert 0101111011.011111_2 to hexadecimal (10)

Sol:

0010	1111	1011	0111	1100
↓	↓	↓	↓	↓
2	F	B	7	C

Result $\Rightarrow 2FB.7C_{16}$

Hexadecimal to Decimal Conversion:

\rightarrow Multiply each digit in the hex number by its position weight and add all those product terms.

EX: ① Convert $5C7_{16}$ to decimal

Sol:

$$\begin{aligned} 5C7 &= (5 \times 16^2) + (12 \times 16^1) + (7 \times 16^0) \\ &= 1280 + 192 + 7 \\ &= 1479_{10} \end{aligned}$$

EX: ② Convert $A0F9.0EB_{16}$ to decimal

Sol:

$$\begin{aligned} A0F9.0EB &= (10 \times 16^3) + (0 \times 16^2) + (15 \times 16^1) + (9 \times 16^0) + (0 \times 16^{-1}) + (14 \times 16^{-2}) \\ &\quad + (11 \times 16^{-3}) \\ &= 41209.0572_{10} \end{aligned}$$

Decimal to Hexadecimal Conversion:

→ Convert integer and fraction parts separately.

→ To convert a decimal integer no.

↳ Divide the given no. by 16 till quotient is 0.

↳ last remainder is MSD.

Hex dabble method

→ To convert a decimal fraction

↳ successively multiply it by 16, till product is 0 or till required accuracy is obtained

↳ The first integer from the top is MSD.

EX: ① Convert 2598.675_{10} to hex.

Sol:

Integer Part

Successive division

16	2598
16	162
16	10
	0

Remainder

Decimal	Hex
6	A26
2	
10	

$\therefore 2598_{10} = A26_{16}$

Fraction Part

Multiply by 16

0.675×16
0.8×16
0.8×16
0.8×16

Remainder

Decimal	Hex
10.8	A
12.8	C
12.8	C
12.8	C

$\therefore 0.675_{10} = 0.Accc_{16}$

∴ Result is $A26.Accc_{16}$

Ex: ② Convert 49056_{10} to binary

Sol: Conversion of very large decimals to binary and

Very large binary nos to decimal is very much simplified if it is done via the hex route

$49056_{10} \rightarrow \text{HEX} \rightarrow \text{BINARY}$

Successive Division

Remainder

16	49056
16	3066
16	191
16	11
	0

Decimal Hex Binary group

0	0	0000
10	A	1010
15	F	1111
11	B	1010

$\therefore 49056_{10} = 1010111110100000_2$

Ex: ③ Convert 1011011101101110_2 to decimal

Sol: Binary \rightarrow Hex \rightarrow decimal

1011	0111	0110	1110
B	7	6	E

$16^3 \ 16^2 \ 16^1 \ 16^0$
B 7 6 E

$(B \times 16^3) + (7 \times 16^2) + (6 \times 16^1) + (E \times 16^0)$
 $= 46958_{10}$

Hexadecimal to Octal Conversion:

→ First Convert hexadecimal no. to binary and then binary no. to octal HEX → BINARY → OCTAL

EX: ① Convert $B9F.AE_{16}$ to octal

Sol: Binary: $1011 \ 1001 \ 1111 \ . \ 1010 \ 1110$
group of 4 bits

→ group of 3 bits: $101 \ 110 \ 011 \ 111 \ . \ 101 \ 011 \ 10$

octal → 5 6 3 7 . 5 3 4

Result ⇒ 5637.534_8

Octal to Hexadecimal Conversion:

OCTAL → BINARY → HEX.

EX: Convert 756.603_8 to hex

Sol:
= group of 3 bits } $111 \ 101 \ 110 \ . \ 110 \ 000 \ 011$

Group of 4 bits: $0001 \ 1110 \ 1110 \ . \ 1100 \ 0001 \ 1000$

Hex: 1 E E . C 1 8

Result ⇒ $1EE.C18_{16}$

* Convert the following nos. with the given radix to decimal and then to binary.

- (a) 4433_5 (b) 1199_{12} (c) 5654_7 (d) 1221_3

Sol:

(a) $4433_5 = 4 \times 5^3 + 4 \times 5^2 + 3 \times 5^1 + 3 \times 5^0$
 $= 618_{10}$

2	618	
2	309	- 0
2	154	- 1
2	77	- 0
2	38	- 1
2	19	- 0
2	9	- 1
2	4	- 1
2	2	- 0
2	1	- 0
	0	- 1 ↑

$618_{10} = 1001101010_2$

Answer

(b) $1199_{12} = 1 \times 12^3 + 1 \times 12^2 + 9 \times 12^1 + 9 \times 12^0$
 $= 1989_{10}$

If decimal no. large, it can be first converted to octal/hex and then to binary

	Decimal	Hex
16 1989		
16 124	- 5	5
16 7	- 7 ↑	C
		7

$1989_{10} = 507_{16} = 010101111000101_2$

Answer

$$(c) \begin{matrix} 5 & 6 & 5 & 4 \\ 7^3 & 7^2 & 7^1 & 7^0 \end{matrix} = 5 \times 7^3 + 6 \times 7^2 + 5 \times 7^1 + 4 \times 7^0$$

$$= 2048_{10} \quad \text{Decimal} \rightarrow \text{Hex} \rightarrow \text{Binary}$$

$$\begin{array}{r|l} 16 & 2048 \\ \hline 16 & 128 - 0 \\ \hline 16 & 8 - 0 \\ \hline & 0 - 8 \uparrow \end{array}$$

$$2048_{10} = \begin{array}{r} 800 \\ / \quad | \quad \backslash \\ 1000 \quad 0000 \quad 0000 \end{array} \quad \text{Answer}$$

$$(d) \begin{matrix} 1 & 2 & 2 & 1 \\ 3^3 & 3^2 & 3^1 & 3^0 \end{matrix} = 1 \times 3^3 + 2 \times 3^2 + 2 \times 3^1 + 1 \times 3^0$$

$$= 57_{10}$$

$$\begin{array}{r|l} 2 & 57 \\ \hline 2 & 26 - 0 \\ \hline 2 & 13 - 0 \\ \hline 2 & 6 - 1 \\ \hline 2 & 3 - 0 \\ \hline 2 & 1 - 0 \\ \hline & 0 - 1 \uparrow \end{array} \quad = 110100 \quad \text{Answer}$$

* Given that $16_{10} = 100_b$, find the value of b .

Sol: Convert 100_b to decimal

$$1 \times b^2 + 0 \times b^1 + 0 \times b^0 = 16 \Rightarrow b^2 = 16 \Rightarrow \boxed{b=4}$$

* Given that $292_{10} = 1204_b$ in some no. system, find the base of that system.

$$\text{Sol: } 292_{10} = 1204_b = 1 \times b^3 + 2 \times b^2 + 0 \times b^1 + 4 \times b^0 = b^3 + 2b^2 + 4$$

By trial & error, $\boxed{b=6}$.

* Binary Arithmetic:

- Binary Addition
- Binary Subtraction
- Binary Multiplication
- Binary Division

Binary Addition:

Rules : $0+0=0$; $0+1=1$; $1+0=1$; $1+1=10$

$1+1+1=11$

↳ result 1 ↳ result 0
 carry 1 carry 1

EX: (1) Add the binary nos. 1011 and 0110

	1 0 1 1	→ 11
+	0 1 1 0	→ 6
	1 0 0 0 1	17

Answer

NOTE: DO not ignore carry

EX: (2) Add the binary nos. 1101.101 and 111.011

	1 1 0 1 . 1 0 1
+	. 1 1 1 . 0 1 1
	1 0 1 0 1 . 0 0 0

Answer

Binary Subtraction:

Rules: $0-0=0$; $1-0=1$; $1-1=0$;

$0-1=1$ with a borrow/

Ex: ① Subtract $(0011)_2$ from $(1100)_2$

$$\begin{array}{r} \overset{1}{1} \overset{1}{1} \overset{0}{0} \overset{0}{0} \rightarrow 12 \\ \ominus \quad 0 \quad 0 \quad 1 \quad 1 \rightarrow 3 \\ \hline 1 \quad 0 \quad 0 \quad 1 \quad \checkmark \\ \hline \end{array}$$

Ex: ②

$$\begin{array}{r} \overset{0}{0} \overset{1}{1} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \\ - \quad \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \\ \hline 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad \checkmark \\ \hline \end{array}$$

~~③~~

$$\begin{array}{r} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \\ - \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \\ \hline 1 \\ \hline 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \\ \hline \end{array}$$

③

$$\begin{array}{r} \overset{1}{1} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{1}{1} \rightarrow 11001011 \\ - \quad \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \\ \hline 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad \checkmark \\ \hline \end{array}$$

④ $1010 \cdot 010$

$$\begin{array}{r} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \\ - \quad \quad \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \\ \hline 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad \checkmark \\ \hline \end{array}$$

⑤

$$\begin{array}{r} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \overset{0}{0} \overset{1}{1} \\ - \quad \quad \quad 1 \quad 1 \quad 1 \quad 1 \\ \hline 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \\ \hline \checkmark \end{array}$$

* Complements of Numbers:

- Complements are used in digital computers to simplify the subtraction operation and for logical manipulation.
- Simplifying operations leads to simpler, less expensive circuits to implement the operations.
- There are 2 types of Complements for each base r system
 - ① radix complement
 - ② diminished radix complement

① Diminished radix complement:

For a Given a no. N with base r having n digits,
 $(r-1)$'s complement of N is its diminished radix complement

→ It is defined as $(r^n - 1) - N$.

EX: Decimal No
 $n=4 \rightarrow$ diminished radix complement is defined as $(10^4 - 1) - N$
 $9999 - N$

EX: → Find diminished radix complement for (a) 546700_{10} (b) 12398

Sol: (a) 9's complement $\Rightarrow (10^6 - 1) - 546700$
 $999999 - 546700$
 $= 453299$

→ Find (b) 9's complement $\Rightarrow (10^5 - 1) - 12398$
 $= 99999 - 12398$
 $= 87601$

For binary nos.

$r=2$, $r-1=1$ \therefore 1's complement is
 diminished radix complement

$n \rightarrow$ no. of digits
 $N \rightarrow$ given no.

\downarrow
 defined as $(2^n - 1) - N$

EX: $n=4$, we have $(2^4 - 1) = 15 = 1111_2$

Thus, 1's complement of a binary no. is obtained by subtracting binary digits from 1.

1's complement is formed by changing 1's to 0's and 0's to 1's.

EX: Find diminished radix complement for the given binary sequence

- (a) 1011000 (b) 0101101

Sol:

$(a) \quad n=7, (2^7 - 1) - N$ $1111111 - 1011000$ $= 0100111$	$(b) \quad n=7, (2^7 - 1) - N$ $1111111 - 0101101$ $= 1010010$
--	--

Diminished Radix Complement

(14).1

↳ For octal nos. → $(r-1)$'s complement
↳ 7's complement

↳ subtract each octal digit from 7

↳ For Hexadecimal nos. → 15's Complement

↳ subtract each hex digit from 15.

(2) Radix Complement:

↳ r 's complement

→ For a given n -digit no. N in base r , it is

defined as → $r^n - N$ for $N \neq 0$
→ 0 for $N = 0$

→ r 's complement is defined as $(r-1)$'s complement

+ 1. i.e.; $[(r^n - 1) - N] + 1$

For decimal no.

↳ radix complement is 10's complement

i.e.; 9's complement + 1

For Binary no.

↳ radix complement is 2's complement

i.e.; 1's complement + 1

Subtraction with Complements: ⁽¹⁵⁾

- 10's Complement
- 9's Complement
- 2's Complement
- 1's Complement

1's Complement, method of subtraction:

Step 1: Obtain 1's Complement of the subtrahend.

Step 2: Add it to the minuend.

Step 3: If there is a carry, bring the carry around and add it to LSB. This is called end around carry. Look at sign bit (MSB).

Step 4: If this is a "0", result is positive and is in true binary form.

Step 4: If MSB is a 1 (or) whether there is no carry, the result is negative and take its 1's complement to (get the magnitude in binary) obtain final result

Ex: Subtract the following binary nos. using 1's complement method

(a) $00011001 - 00001110$

(b) $00001110 - 00011001$

Sol: (a) Minuend \rightarrow 0 0 0 1 1 0 0 1

1's comple Subtrahend \rightarrow 1 1 1 1 0 0 0 1
+
1 1 1 1

1 0 0 0 0 1 0 1 0
+ \rightarrow 1 (end around carry)

0 0 0 0 1 0 1 1
MSB LSB

MSB = 0

\therefore Result is positive & is in pure binary

Answer: 00001011 $+11_{10}$

(b) Minuend \rightarrow 0 0 0 0 1 1 1 0

Subtrahend \rightarrow 1 1 1 0 0 1 1 0 (1's complement)

+
1 1 1 1
1 1 1 1 0 1 0 0
MSB LSB (No carry)

No carry
 \rightarrow MSB = 1

So result is negative

1's complement

Answer: 00001011 -11_{10}

2's Complement method of subtraction:

Step 1: Obtain 2's complement of the subtrahend.

Step 2: Add it to the minuend.

Step 3: If there is a carry, Ignore it Look at sign bit (MSB). If this is a "0", result is positive and is in true binary form.

Step 4: If MSB is a 1 ^{if} there is no carry, the result is negative and take its 2's complement to (find its magnitude in binary) obtain final result.

EX: Given two binary nos. $X = 1010100$ and $Y = 1000011$, perform the subtraction (a) $X - Y$ and (b) $Y - X$ by using 2's complements.

Sol: (a) Minuend $\rightarrow 1010100$
 Subtrahend $\rightarrow 0111101$ (2's complement of Y)

$$\begin{array}{r}
 1010100 \\
 + 0111101 \\
 \hline
 1001001
 \end{array}$$
 Ignore $\textcircled{1}$ MSB 001001 LSB

MSB = 0 \rightarrow so result is positive

001000 | Answer

(b) Minuend \rightarrow 1 0 0 0 0 1 1
 Subtrahend \rightarrow 0 1 0 1 1 0 0 (2's complement of X)

$$\begin{array}{r}
 \\
 \hline
 1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \\
 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \\
 \hline
 \end{array}$$

MSB LSB

No carry

MSB=1 \rightarrow so, result is negative

2's complement

0 0 1 0 0 0 1 Answer

EX: Subtract 00001101 from 00101110 using 2's complement

Sol:

Minuend \rightarrow 0 0 1 0 1 1 1 0
 Subtrahend \rightarrow 1 1 1 1 0 0 1 1 (2's complement of Y)

$$\begin{array}{r}
 \\
 \hline
 0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0 \\
 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \\
 \hline
 \end{array}$$

Ignore 1 MSB

MSB=0 \rightarrow result is positive

0 0 1 0 0 0 0 1 Answer

* Signed Binary Numbers:

- Unsigned numbers represents positive integers (including zero).
- ~~Signed~~ In ordinary arithmetic, a negative no. is indicated by a minus sign & a positive no. by a plus sign.
- Because of hardware limitations, computers must represent everything with binary digits.
- There are two ways of representing
- For Unsigned nos. → left most bit is MSB of the no.

For signed nos. → left most bit represents the sign & rest of the bits represent the no.

If sign bit is 0 → positive
 1 → negative

EX: ① +9 (unsigned binary) → 01001

↳ signed binary → 01001
 sign bit

② -9 → signed binary → 11001

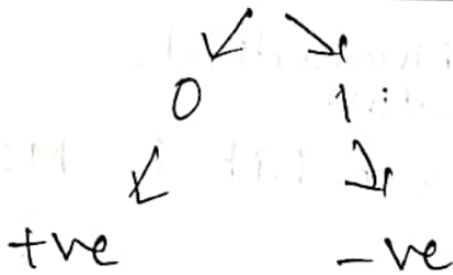
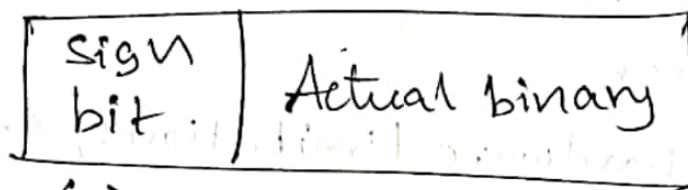
Representation of signed numbers :

① signed-magnitude form

② complement form $\begin{cases} 1's \\ 2's \end{cases}$

①

Syntax:



unsigned \rightsquigarrow 9 \rightsquigarrow 1001

signed \rightsquigarrow +9 \rightsquigarrow 01001

\rightsquigarrow -9 \rightsquigarrow 11001

EX: ① Represent -26 in signed-magnitude form

Sol:

$(26)_{10} \rightarrow ()_2$
 $\hookrightarrow (11010)_2$

2	26	
2	13	— 0
2	6	— 1
2	3	— 0
2	1	— 1
2	0	— 1 \uparrow

signed-magnitude representation

of -26 is $\underline{1}11010$

sign
bit

+26 \rightarrow 011010

sign
bit

Ex: Represent -26 in complement form

Sol: $(26)_{10} = (11010)_2$ SMR $\rightarrow -26$
 $\rightarrow 111010$

\Rightarrow 1's Complement form of -26 \rightarrow $\underline{1}00101$
(sign bit as it is, take 1's complement of actual binary)
sign bit

\Rightarrow 2's Complement form of -26 \rightarrow $\underline{1}00110$
(sign bit as it is, obtain 2's complement of actual binary)
sign bit

Ex: (2) Represent +26 in all forms

Sol: Binary of 26 $\rightarrow 11010$

SMR of +26 $\rightarrow 011010$

1's compl. of +26 \rightarrow ~~011010~~ ~~000101~~ 011010

2's Compl. of +26 \rightarrow ~~011010~~ ~~000110~~ 011010

EX: Find 2's complement of given signed no.

1 00110.

Sol: $\overline{1} 00110$
 Sign bit

1's Compl $\rightarrow 1 11001$

2's comple $\rightarrow \begin{array}{r} 1 11001 \\ + \\ \hline 1 11010 \end{array} \rightarrow -26 \text{ Answer}$
 Sign bit

NOTE: 1's complement of (1's complement of x) = x
 2's complement of (2's complement of x) = x

EX: Express -45 in 8-bit 2's complement form

Sol: $(45)_{10} = (?)_2$

2		45	
2		22	-1
2		11	-0
2		5	-1
2		2	-1
2		1	-0
		0	-1 ↑

$(101101)_2$

SMR of +45 = 00101101

SMR of -45 = 10101101

1's comple $\rightarrow 11010010$

2's comple $\rightarrow \begin{array}{r} 11010010 \\ + 1 \\ \hline 11010011 \end{array} \text{ Answer}$

Arithmetic addition:

For Negative nos. \rightarrow ^{Take} 2's complement

Case (1): Both positive

Ex: ① Add $(6)_{10}$ and $(13)_{10}$ using 8-bit.

$$\begin{array}{r}
 +6 \longrightarrow 00000110 \\
 +13 \longrightarrow 00001101 \\
 \hline
 +19 \longrightarrow 00010011
 \end{array}$$

Carry bit need to be discarded.

② Add

Case (2): Smaller Negative

Ex: ① Add $(13)_{10}$ and $(-6)_{10}$ using 8-bit

$$\begin{array}{r}
 \text{Sol: } 13 \longrightarrow 00001101 \\
 -6 \xrightarrow{2's\ comp} 11111010 \\
 \hline
 +7 \longrightarrow 00000111 \text{ Answer}
 \end{array}$$

Ignore carry

$$\begin{array}{r}
 (+6)_{10} = 00000110 \\
 (-6)_{10} = 10000110 \xrightarrow{2's} 11111001 \\
 \hline
 +1 \xrightarrow{2's} 11111010
 \end{array}$$

② Add $(46)_{10}$ and $(-14)_{10}$ using 8-bit.

Sol:
$$\begin{array}{r} 2 \mid 46 \\ \hline 2 \mid 23 - 0 \\ \hline 2 \mid 11 - 1 \\ \hline 2 \mid 5 - 1 \\ \hline 2 \mid 2 - 1 \\ \hline 2 \mid 1 - 0 \\ \hline 0 - 1 \uparrow \end{array}$$

$(46)_{10} = (101110)_2$

~~$(-14)_{10}$~~

$+46_{10} = 00101110$

SMR $\rightarrow -14_{10} = 10001110$
Sign bit

$(14)_{10} = 00001110$

1's comple. $\rightarrow 11110001$

2's comple. $\rightarrow \begin{array}{r} 11110001 \\ + 1 \\ \hline 11110010 \end{array}$

$$\begin{array}{r} +46 \rightarrow 00101110 \\ -14 \rightarrow 10001110 \\ \hline +32 \rightarrow 10000000 \end{array}$$

carry ignore sign bit

Answer

Case (3): Greater Negative

EX: ① Add $(-13)_{10}$ and $(6)_{10}$ using 8-bit.

Sol: $(13)_{10} \rightarrow 00001101$

SMR $\rightarrow -13_{10} \rightarrow 10001101$

1's comple. $\rightarrow 11110010$

2's comple. $\rightarrow 11110011$

$+6 \rightarrow 00000110$

$-13 \rightarrow 11110011$

$-7 \rightarrow 11111001$

Sign bit

↓ No carry

Take 2's comple.

000011 Answer

Case (4): Both negative

Ex: ① Add -25 to -14 using 8-bit

Sol:



$$\begin{array}{r}
 2 \overline{) 25} \\
 \underline{20} \\
 5 \\
 \underline{4} \\
 1 \\
 \underline{0} \\
 0
 \end{array}$$

$$25_{10} = 11001_2$$

$$25_{10} \rightarrow 8\text{-bit} \rightarrow 00011001$$

$$+25_{10} \rightarrow 00011001$$

sign bit

$$\text{SMR } -25_{10} \rightarrow \underline{1}0011001$$

sign bit

$$\begin{array}{l}
 -25_{10} \left\{ \begin{array}{l}
 1's \text{ complement} \rightarrow 11100110 \\
 2's \text{ complement} \rightarrow 11100111
 \end{array} \right.
 \end{array}$$

$$\Rightarrow 14_{10} = 00001110$$

$$\text{SMR } -14_{10} = \underline{1}0001110$$

$$1's \text{ complement} \rightarrow 11110001$$

$$2's \text{ complement} \rightarrow 11110010$$

$$\begin{array}{r}
 -25 \xrightarrow{2's} 11100111 \\
 -14 \xrightarrow{2's} 11110010 \\
 \hline
 -39 \longrightarrow 111011001
 \end{array}$$

Carry ignore

2's complement

$$\text{sign bit } \boxed{10100111} \text{ Answer}$$

② Add -75 to $+26$ using 8-bit 2's complement arithmetic.

Sol:
$$\begin{array}{r} 2 \overline{) 26} \\ \underline{2 13} \\ 2 6 \\ \underline{2 3} \\ 2 1 \\ \underline{ 0} \uparrow \end{array}$$

$26_{10} = 11010$

$(+26)_{10} \rightarrow 8\text{-bit}$

00011010

$$\begin{array}{r} 16 \overline{) 75} \\ \underline{16 4} \text{B} \\ 0 \uparrow 4 \end{array}$$

$(75)_{10} \rightarrow \text{Hex} \rightarrow \text{Binary}$
 $4\text{B} \rightarrow 01001011$
 $+75_{10} \rightarrow 01001011$

$\text{SMR} \rightarrow -75_{10} \rightarrow 11001011$
Sign bit

1's complement $\rightarrow 10110100$

2's complement $\rightarrow 10110101$

$$\begin{array}{r} -75 \xrightarrow{2's} 10110101 \\ +26 \rightarrow 00011010 \\ \hline -49 \rightarrow 1 \underline{1001111} \end{array}$$

Sign bit

↓ No carry

Take 2's complement of actual binary

$$1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ \text{Answer}$$

$2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0$

Arithmetic Subtraction:

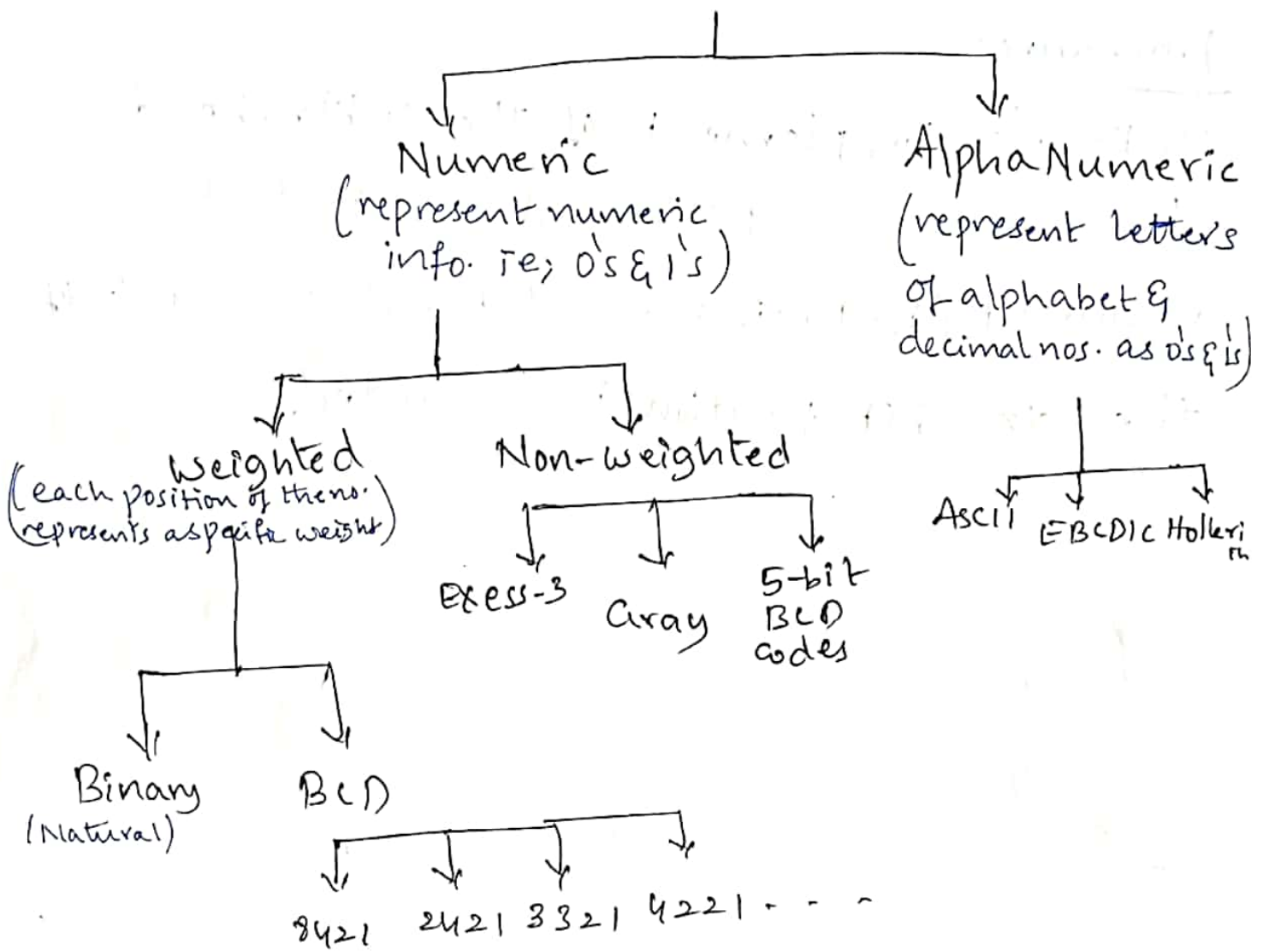
(2)

Procedure:

Take 2's complement of the subtrahend (including sign bit) and add it to the minuend (including sign bit). A carry out of the sign-bit position is discarded.

* Binary codes:

Codes



→ Natural Binary code:

• An n-bit binary code is a group of n bits that assumes upto 2^n distinct combinations of 1's and 0's, with each combination representing one element of the set that is being coded.

- A set of 4 elements, requires a two-bit code
- " 8 " " " a 3-bit code
- " 16 " " " " 4 " "

→ BCD Code:

- A BCD code is one, in which the digits of a decimal no are encoded - one at a time - into groups of four binary digits.
- These codes combine the features of decimal and binary nos.

Code word: It is defined as the sequence of binary digits which represents a decimal digit

- BCD is a weighted code
- Widely used BCD code is 8421.

NOTE: Binary combinations from 1010 to 1111 are not used & have no meaning in BCD.

Decimal symbol	BCD digit
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Advantage of BCD code:

- ① Using BCD code, ^{its easy to} ~~conversion~~ convert to and from decimal.
- ② less efficient than the pure binary, in the sense that it requires more bits.

For ex, 14 → 1110 (pure binary)
 ↳ 0001 0100 (BCD)

Disadvantage of BCD code:

- ① Arithmetic operations are more complex than they are in pure binary.
- ② The rules of binary addition & subtraction do not apply to the entire 8421 number, but only to the individual 4-bit groups.

BCD Addition:

Step 1: Individually add the corresponding digits of the decimal nos. expressed in 4-bit binary groups starting from LSD.

Step 2: If there is a carry out of one group to the next group, it is added to the next group.

(or) If the sum term is an illegal code, then add 6_{10} (0110) to the sum term of that group.

Step 3: If there is no carry, and ^{if} the sum term is not an illegal code, no correction is needed.

Perform the following decimal additions

EX: (1) in the 8421 code

(a) $25 + 13$ (b) $679.6 + 536.8$

Sol:

(a)

25	→	<u>B_{CD}</u>	0010 0101
+ 13	→		0001 0011
38			0011 1000
			(No carry & sum no illegal code)
			↓ ↓
			3 8

(b)

679.6	→	<u>B_{CD}</u>	0110 0111 1001 . 0110
+ 536.8	→		0101 0011 0110 . 1000
1216.4			↓ 0111 1010 1111 . 1110
			(No carry, but illegal 0110 codes)

(Propagate carry)

0001	+	0010	+	0001	+	0110	+	0100	0
+ 1	←	+ 1	←	+ 1	←	+ 1	←	+ 1	←
0001		0010		0001		0110		0100	0

1 2 1 6 4

(c)

4	→	0100
+ 5	→	0101
9		1001

(d)

4	→	0100
+ 8	→	1000
12		1100 (illegal code)
		+ 0110
		10010

(e) $8 \rightarrow 1000$
 $+ 9 \rightarrow 1001$

 $27 \quad 10001$ (illegal code)
 $+ 0110$

 10110

 $1 \quad 7$ ✓

(f) $184 + 576$

$184 \rightarrow$	0001	1000	0100
$+ 576 \rightarrow$	0101	0111	0110
760	0110	1111	1010
		0110	0110
		11	11
	0110	0010	1000
	$+K$	$+1$	
	$0111 \quad 0110 \quad 0000$		
	7	6	0

BCD subtraction:

Step 1: Subtract the digits of each 4-bit group of the subtrahend from the corresponding 4-bit group of the minuend in binary starting from the LSD.

Step 2: If there is a borrow from the next group, then subtract $6_{10} (0110)$ from the difference term of this group.

Step 3: If there is no borrow from the next 21

higher group then no correction is required

EX: perform the following decimal subtractions in the 8421 BCD code.

(a) $38 - 15$ (b) $206.7 - 147.8$

Sol:

(a)
$$\begin{array}{r} 38 \\ - 15 \\ \hline 23 \end{array}$$

$$\begin{array}{r} 0011 \quad 1010 \quad (38 \text{ in BCD}) \\ - 0001 \quad 0101 \quad (15 \text{ in BCD}) \\ \hline 0110 \quad 0011 \quad (\text{No borrow}) \\ \quad \quad 2 \quad 3 \quad \checkmark \end{array}$$

(b)
$$\begin{array}{r} 206.7 \\ - 147.8 \\ \hline 58.9 \end{array}$$

$$\begin{array}{r} 0010 \quad 0100 \quad 0111 \quad 1011 \quad (206.7) \\ - 0001 \quad 0100 \quad 0111 \quad 1000 \quad (147.8) \\ \hline 0000 \quad 1011 \quad 1110 \quad 1111 \\ - 0110 \quad 0110 \quad 0110 \\ \hline 0000 \quad 0101 \quad 1000 \quad 1001 \\ \quad \quad 5 \quad 8 \quad . \quad 9 \quad \checkmark \end{array}$$

BCD subtraction using 9's complement Method:

① Perform the following decimal subtractions in BCD by 9's Complement method.

(a) $305.5 - 168.8$ (b) $679.6 - 885.9$

Sol: (a) $305.5 \rightarrow 305.5$
 $-168.8 \rightarrow +831.1$ (9's complement)

 136.7

 ① 136.6
 $\hookrightarrow +1$ (End-around carry)

 136.7

$305.5_{10} \rightarrow 0011\ 0000\ 0101 \cdot 0101$
 $+831.1_{10} \rightarrow +1000\ 0011\ 0001 \cdot 0001$

$1011\ 0011\ 0110 \cdot 0110$ (No carry, 1011 illegal code)
 $+0110$

① $0001\ 0011\ 0110 \cdot 0110$
 $\hookrightarrow +1$

$0001\ 0011\ 0110 \cdot 0111$
 $1\ 3\ 6 \cdot 7$

BCD Subtraction using 9's complement Method: (24) ①

Step 1: Take 9's complement of the subtrahend

Step 2: Add it to the minuend using BCD addition

Step 3: If the result is invalid BCD, then correct by adding 6.

Step 4: shift the carry to next bits

Step 5: If end around carry is generated then add it to the result

If there is no carry, it indicate the answer is -ve, & result is its 9's complement. Take 9's complement again ~~and~~ to get actual result

BCD Subtraction using 10's complement Method:

- Step 1: Take 10's complement of subtrahend.
 - Step 2: Add it to minuend using BCD addition.
 - Step 3: If the result is invalid BCD, then correct it by adding 6.
 - Step 4: Shift the carry to next bit.
 - Step 5: If end around carry is present, discard it. If not, say result is -ve.
- If result is in 10's complement form, take 10's complement again to get actual result.

$$\begin{array}{r}
 (b) \quad 679.6 \rightarrow 679.6 \\
 - 885.9 \rightarrow 114.0 \text{ (9's complement)} \\
 \hline
 -206.3 \qquad \qquad \qquad 793.6 \\
 \qquad \qquad \qquad \qquad \qquad \downarrow \text{9's complement} \\
 \qquad \qquad \qquad \qquad \qquad -206.3
 \end{array}$$

$$\begin{array}{l}
 679.6 \rightarrow 0110 \ 0111 \ 1001 \ . \ 0110 \\
 114.0 \rightarrow 0001 \ 0001 \ 0100 \ . \ 0000
 \end{array}$$

$$\begin{array}{r}
 111 \\
 \hline
 0111 \ 1000 \ 1101 \ . \ 0110 \\
 + 0110
 \end{array}$$

$$\begin{array}{r}
 1 \\
 \hline
 0111 \ 1000 \ 0011 \ . \ 0110 \\
 14
 \end{array}$$

$$\begin{array}{r}
 \hline
 0111 \ 1001 \ 0011 \ . \ 0110 \\
 7 \quad 9 \quad 3 \quad . \quad 6
 \end{array}$$

There is no carry & therefore result is -ve and is in its 9's complement form. The 9's complement of 793.6 is 206.3. So the result is -206.3

BCD Subtraction using 10's Complement Method:

Ex: Perform the following decimal subtractions in BCD by 10's complement method.

(a) $342.7 - 108.9$ (b) $206.4 - 507.6$

Sol: (a) $342.7 \rightarrow 342.7$
 $- 108.9 \rightarrow +891.1$ (10's complement)

 233.8 $\textcircled{1}233.8$ (Ignore carry)

$342.7 \rightarrow 0011\ 0100\ 0010\ .\ 0111$
 $+ 891.1 \rightarrow 1000\ 1001\ 0001\ .\ 0001$
+

 $1011\ 1101\ .\ 0111\ 1000$
 $+ 0100\ 0110$

 $\textcircled{1}0001\ \textcircled{1}0011\ 0011\ .\ 1000$
+

 $\textcircled{1}0000\ 0011\ 0011\ .\ 1000$
Ignore carry \leftarrow 2 3 3 . 8 ✓

(b) $206.4 \rightarrow 206.4$
 $- 507.6 \rightarrow +492.4$ (10's complement)

 -301.2 698.8 (No carry)
↓ 10's

 -301.2 ✓

$$\begin{array}{r}
 206.4 \rightarrow 0010\ 0000\ 0110.0100 \\
 +492.4 \rightarrow 0100\ 1001\ 0010.0100 \\
 \hline
 0110\ 1001\ 1000.1000 \text{ (No carry, No illegal)} \\
 \hline
 \text{6} \quad \text{9} \quad \text{8} \quad \text{.8} \\
 \hline
 \text{0110} \quad \text{1111} \quad \text{1000} \quad \text{1000}
 \end{array}$$

There is no carry & result is -ve and is in its 10's complement form. The 10's complement of 698.8 is 301.2 so the result is -301.2

Excess-3 code:

- Unweighted code. Also called as XS-3
- Each coded combination is obtained from the corresponding binary value plus 3.
- It's an example of self-complementing code

Decimal Digit	Excess-3
0	0011
1	0100
2	0101
3	0110
4	0111
5	1000
6	1001
7	1010
8	1011
9	1100

if the codeword of 9's complement of N is $9-N$ can be obtained from the code word of N by interchanging all the 0's and 1's.

2421 Code :

- Example of ~~an~~ self-complementing code
- It is a weighted code

Decimal Digit	2421
0	0000
1	0001
2	0010
3	0011
4	0100
5	1011
6	1100
7	1101
8	1110
9	1111

Gray code :

- A/D converter \rightarrow Analog to digital
- Non-weighted code, Not suitable for arithmetic operations

1100	0
1010	1
1011	2
0110	3
1110	4
0001	5
1001	6
0101	7
1101	8
0011	9