

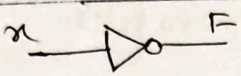
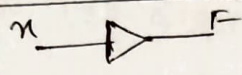
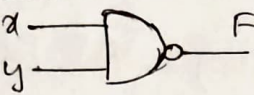
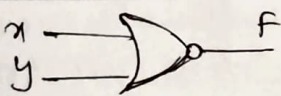
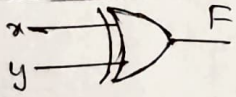
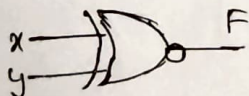


• Digital Logic Gates (Algebraic Simplification of Digital Logic Gates)

- Since Boolean functions are expressed in terms of AND, OR, and NOT operations, it is easier to implement a Boolean function with these types of gates.
- Still there is a possibility of constructing gates for the other logic operations.
- Factors to be considered for construction of other types of logic gates are:
 - 1) the feasibility and economy of producing the gate with physical components.
 - 2) the possibility of extending the gate to more than two inputs
 - 3) the basic properties of the binary operator, such as commutativity and associativity.

4) the ability of the gate to implement Boolean functions alone or in conjunction with other gates.

Name	Graphic Symbol	Algebraic Function	Truth Table															
AND		$F = x \cdot y$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	x	y	F	0	0	0	0	1	0	1	0	0	1	1	1
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OR		$F = x + y$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	x	y	F	0	0	0	0	1	1	1	0	1	1	1	1
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Inverter		$F = x'$	<table border="1"> <thead> <tr> <th>x</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>0</td></tr> </tbody> </table>	x	F	0	1	1	0									
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Buffer		$F = x$	<table border="1"> <thead> <tr> <th>x</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td></tr> </tbody> </table>	x	F	0	0	1	1									
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NAND		$F = (xy)'$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	x	y	F	0	0	1	0	1	1	1	0	1	1	1	0
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NOR		$F = (x + y)'$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	x	y	F	0	0	1	0	1	0	1	0	0	1	1	0
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Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	x	y	F	0	0	0	0	1	1	1	0	1	1	1	0
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Exclusive-NOR (XNOR) or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$ $= x \odot y$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	x	y	F	0	0	1	0	1	0	1	0	0	1	1	1
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• Exclusive-OR Function (Properties of XOR Gates)

→ The exclusive-OR (XOR), denoted by the symbol \oplus , is a logical operation that performs the Boolean operation: $x \oplus y = xy' + x'y$

→ The exclusive-OR is equal to 1 if only x is equal to 1 or if only y is equal to 1 (i.e., x and y differ in value), but not when both are equal to 1 or when both are equal to 0

→ The exclusive-NOR (XNOR), also known as equivalence performs the Boolean operation: $(x \oplus y)' = xy + x'y'$

→ The exclusive-NOR is equal to 1 if both x and y are equal to 1 or if both are equal to 0.

→ The following identities apply to the exclusive-OR operation:

$$x \oplus 0 = x$$

$$x \oplus 1 = x'$$

$$x \oplus x = 0$$

$$x \oplus x' = 1$$

$$x \oplus y' = x' \oplus y = (x \oplus y)'$$

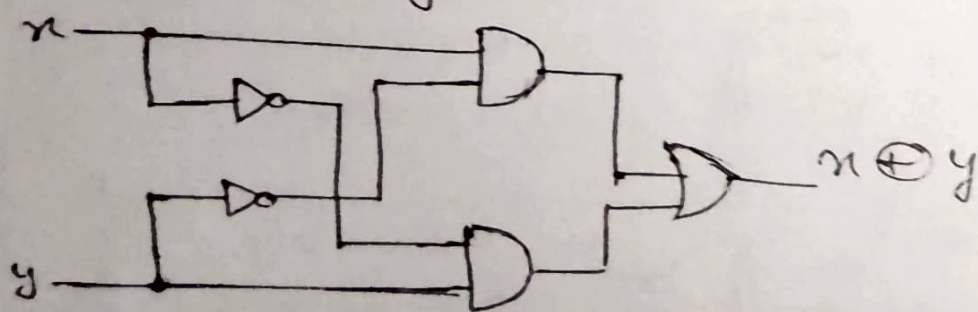
→ Any of these identities can be proven with a truth table or by replacing the \oplus operation by its equivalent Boolean expression.

→ The exclusive-OR operation is both commutative and associative.

$$A \oplus B = B \oplus A$$

$$(A \oplus B) \oplus C = A \oplus (B \oplus C) = A \oplus B \oplus C$$

→ A two-input exclusive-OR function is constructed with conventional gates using two inverters, two AND gates, and an OR gate.



→ The implementation of the exclusive-OR with four NAND gates is as follows;

$$(x' + y')x + (x' + y)y = xy' + x'y = x \oplus y$$

