- Quine-McCluskey (QM) Technique (Tabular Method)

→ K-map is the convenient and effective in simplifying a Boolean expression.
→ However, for increased number of input variables, QM method can be adopted.
→ QM method is a tabular method of minimization.
→ The basic Quine-McCluskey procedure is as follows:

1) Find the prime implicants of the function
2) Construct the prime-implicant table and find the essential prime implicants (essential rows) of the function.
3) Include the essential prime implicants in the minimal sum.
4) After all essential prime implicants are deleted from the prime-implicant table, determine the dominated rows and dominating columns in the table, delete all dominated rows and dominating columns, and find the (secondary) essential prime implicants.
5) Repeat step 3 and 4 as many times as they are applicable until a minimal cover of the function is found.

Q.) Consider the minimization of the following switching function using the Quine-McCluskey method.

$$f(x_1, x_2, x_3, x_4) = \sum_m (0,5,7,8,9,10,11,14,15)$$

1) Find the prime implicants of the function.

(a) Represent each term (minterm) of the canonical sum-of-products form by a binary code.

(b) Find the decimal number for each binary.

(c) Define the number of 1's in binary number as the index of the number. Group all the binary numbers of the same index into a group. List all the groups in a column in the index-ascending order.

(d) Start with the terms in the set of lowest index; compare them with those, if any, in the set whose index is 1 greater, and eliminate all redundant variables.

(e) Check off all the terms that entered into the combination. The ones that are left are prime implicants from which a minimum sum is to be selected.

f)

	Decimal Number	Binary Representation of Each Term	Decimal Numbers	First Reduction	Decimal Numbers	Second Reduction
Index 0	0	0000✓	0,8	_000 E	8,9,10,11	10_ _ B
Index 1	8	1000✓	8,9	100_✓	10,11,14,15	1_1_ A
Index 2	5	0101✓	8,10	10_0✓		
	9	1001✓	5,7	01_1 D		
	10	1010✓	9,11	10_1✓		
Index 3	7	0111✓	10,11	101_✓		
	11	1011✓	10,14	1_10✓		
	14	1110✓	7,15	_111 C		
Index 4	15	1111✓	11,15	1_11✓		
			14,15	111_✓		

(f) Repeat steps (d) and (e) until no further reduction is possible, thereby we obtain the set of all prime implicants each of which is designated by a capital English letter.

2) Construct the prime-implicant table and find the essential prime implicants of the function.

(a) Construct a table in which each column carries a decimal number at the top which corresponds to one of the minterms in the canonical sum-of-products form in the given function. The columns are assigned by such a number in ascending order. Each row corresponds to one of the prime implicants designated by A, B, C, D, and E at the left.

(b) Make a cross under each decimal number that is a term contained in the prime implicant represented by that row.

(c) Find all the columns that contain a single cross and a circle in them; place an asterisk to the left of those rows in which you have circled a cross. The rows marked with an asterisk are the essential prime implicants.

3) Include the essential prime implicants in the minimal sum.

Prime Implications	Minterms 0	5	7	8	9	10	11	14	15
*A						×	×	⊗	×
*B				×	⊗	×	×		
C			×						×
*D		⊗	×						
*E	⊗			×					

$$f(x_1, x_2, x_3, x_4) = A + B + D + E$$
$$= x_1 x_3 + x_1 x_2' + x_1' x_2 x_4 + x_2' x_3' x_4'$$