Sample Problems in Predicate Logic

1. Voting.
   Given the following clues,

   (a) Everyone who is 18 years or older is eligible to vote.

   (b) No one graduates from high school unless he/she is at least 18 years old.

   (c) Sarah attended the high school graduation ceremony last week.

   (d) Anyone who completes the high school graduation requirements is considered to be graduated from high school.

   (e) A high school student can attend the graduation ceremony only if the student has completed high school graduation requirements.

   derive a reasonable conclusion.

   Rewrite the statements so that they can be easily transformed into predicates:

   (a) If a person is 18 years or older, then he/she is eligible to vote.

   (b) A person does not graduate from high school if he/she is not at least 18 years old.

   (c) Sarah is a person and she attended the high school graduation ceremony last week.

   (d) If a person completes high school graduation requirements, then he/she is considered to be graduated from high school.

   (e) if a high school student attends the graduation ceremony, then the student has completed high school graduation requirements.

   Now rewrite them using predicate logic notation:

   $$\forall\, p : Person \bullet 18\, YearsOrOlder(p) \Rightarrow eligibleToVote(p) \quad (1)$$
   $$\forall\, p : Person \bullet \neg\, 18\, YearsOrOlder(p) \Rightarrow \neg\, graduateFromHighSchool(p) \quad (2)$$
   $$\exists\, p : Person \bullet p = Sarah \wedge attendGraduationCeremony(p) \quad (3)$$
   $$\forall\, p : Person \bullet completesGraduationRequirements(p) \Rightarrow graduateFromHighSchool(p) \quad (4)$$
   $$\forall\, p : Person \bullet attendGraduationCeremony(p) \Rightarrow completesGraduationRequirements(p) \quad (5)$$

   Using (3) and One-point rule, infer
   $$attendGraduationCeremony(Sarah) \quad (6)$$
   Using (6), (5) and Modus Ponens rule, infer
   $$completesGraduationRequirements(Sarah) \quad (7)$$
   Using (7), (4) and Modus Ponens rule, infer

$graduateFromHighSchool(Sarah)$ (8)

Now, rewrite (2) using Modus Tollens rule as

$\forall\, p : Person \bullet graduateFromHighSchool(p) \Rightarrow 18\, YearsOrOlder(p)$ (9)

Using (9), (8) and Modus Ponens rule, infer

$18\, YearsOrOlder(Sarah)$ (10)

Using (10), (1) and Modus Ponens rule, infer

$eligibleToVote(Sarah)$ (11)

Therefore, the conclusion is "Sarah is eligible to vote."

2. Insurance Policy.
Given the following information,

   (a) Home owners are required to take home insurance.

   (b) Home owners are required to take flood insurance only if the home is close to water.

   (c) Not everyone takes auto insurance except owners of vehicles.

   (d) If a person takes home insurance and auto insurance, he/she does not need to take flood insurance.

   (e) Shawn owns a home near a lake and has a car.

   draw a reasonable conclusion.

   Rewrite the statements so that they can be easily transformed into predicates:

   (a) If a person is a home owner, he/she is required to take home insurance.

   (b) If a person is a home owner and is required to take flood insurance, then it is evident (or clear) that the home of the person is close to water.

   (c) If a person owns a vehicle, then person takes auto insurance.

   (d) If a person takes home insurance and auto insurance, he/she does not need to take flood insurance.

   (e) Shawn is a person and he is a home owner and his home is close to water and he owns a vehicle.

   Now rewrite them in predicate logic notation:

$$\forall\, p : Person \bullet homeOwner(p) \Rightarrow takesHomeInsurance(p) \quad (1)$$
$$\forall\, p : Person \bullet homeOwner(p) \land takesFloodInsurance(p) \Rightarrow homeCloseToWater(p) \quad (2)$$
$$\forall\, p : Person \bullet OwnsVehicle(p) \Rightarrow takesAutoInsurance(p) \quad (3)$$
$$\forall\, p : Person \bullet takesHomeInsurance(p) \land takesAutoInsurance(p) \Rightarrow \neg\, takesFloodInsurance(p) \quad (4)$$
$$\exists\, p : Person \bullet p = Shawn \land homeOwner(p) \land homeCloseToWater(p) \land OwnsVehicle(p) \quad (5)$$

   Using (5) and One-point rule, infer
$$homeOwner(Shawn) \land homeCloseToWater(Shawn) \land OwnsVehicle(Shawn) \quad (6)$$
   Using (5) and Conjunctive simplification rule (from Propositional logic), infer
$$homeOwner(Shawn) \quad (7)$$
$$homeCloseToWater(Shawn) \quad (8)$$
$$OwnsVehicle(Shawn) \quad (9)$$
   Using (9), (3) and Modus Tollens, infer $takesAutoInsurance(Shawn)$ (10)
   Using (7), (1) and Modus Ponens, infer $takesHomeInsurance(Shawn)$ (11)

Using (10), (11) and Conjunctive addition (from Propositional logic), infer
$takesHomeInsurance(Shawn) \wedge takesAutoInsurance(Shawn)$ (11)
Using (11), (4) and Modus Ponens, infer $\neg\ takesFloodInsurance(Shawn)$ (12)

Therefore, the conclusion is "Shawn does not take flood insurance."

3. What animals do?.
   Given the following facts

   (a) I trust every animal that belongs to me.

   (b) Dogs gnaw bones.

   (c) I admit no animals into my study room unless they will beg me.

   (d) All animals in the yard are mine.

   (e) I admit every animal that I trust into my study room.

   (f) The only animals that really beg me are dogs.

   What is the conclusion?

   Rewrite the clues into the form that can be easily translated into predicates:

   (a) If an animal belongs to me, I trust it.

   (b) If an animal is a dog, it gnaws bones.

   (c) I admit an animal into my study room only if it will beg me.
   This statement can be rewritten in the form,
   If I admit an animal into my study room, it should have begged me.

   (d) If an animal is in the yard, it is mine.

   (e) If I trust an animal, I admit it into my study room.

   (f) If an animal is not a dog, it does not beg me.
   This can also be written in the form,
   If an animal has begged me, it should be a dog.

   Now, write the predicates.

$$\forall\, a : Animal \bullet belongsToMe(a) \Rightarrow I - trust(a) \quad (1)$$
$$\forall\, a : Animal \bullet dog(a) \Rightarrow gnawsBones(a) \quad (2)$$
$$\forall\, a : Animal \bullet admitIntoStudyRoom(a) \Rightarrow hasBeggedMe(a) \quad (3)$$
$$\forall\, a : Animal \bullet inTheYard(a) \Rightarrow belongsToMe(a) \quad (4)$$
$$\forall\, a : Animal \bullet I - trust(a) \Rightarrow admitIntoStudyRoom(a) \quad (5)$$
$$\forall\, a : Animal \bullet hasBeggedMe(a) \Rightarrow dog(a) \quad (6)$$

   Using (4), (1) and Modus Ponens, infer $\forall\, a : Animal \bullet inTheYard(a) \Rightarrow I - trust(a)$ (7)
   Using (7), (5) and Modus Ponens, infer
$$\forall\, a : Animal \bullet inTheYard(a) \Rightarrow admitIntoStudy(a) \quad (8)$$
   Using (8), (3) and Modus Ponens, infer
$$\forall\, a : Animal \bullet inTheYard(a) \Rightarrow hasBeggedMe(a) \quad (9)$$
   Using (9), (6) and Modus Ponens, infer

$\forall\, a : Animal \bullet inTheYard(a) \Rightarrow dog(a)$ (10)
Using (10), (2) and Modus Ponens, infer
$\forall\, a : Animal \bullet inTheYard(a) \Rightarrow gnawBones(a)$ (11)

Therefore, the conclusion is "All animals in the yard gnaw bones."